03/07/18

Discrete Mathematics

UNIT-I

1. Logic & proofs

Propositions (statement)

A proposition is a declarative sentence which is either true or false but not both.

ex

Delhi is the Capital of India - T

$3+4=7$ - T

Notations :-

If a proposition is true then its truth value is denoted by T. If it is false then its truth value is denoted by F.

P, Q, R, S ..... are used to denote proposition.

The following sentences are not proposition

1. what is the height of Himalaya?
2. what a wonderful joke is this!
3. obey my orders.
4. $x+5=9$

Primary Statement : (Atomic Statement)

A declarative Sentence which cannot be further split up into simple sentence is called primary statement.

Compound statement:

statement which contain one or more primary statements and some connectivities are called Compound Statement.

Conjuction :- ($\wedge$(and)) :-

If both p and q ~~are~~ have truth values T, then $p \wedge q$ has the truth value T. otherwise $p \wedge q$ has the truth value F.

Ex:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex:

Find the symbolic form ~~where~~ when it is raining and I am getting cold.

Solu

p: It is raining

q: I am getting cold

$p \wedge q$: It is raining and I am getting cold.

Disjunction: ($\vee$) (or)

$p \vee q$ has the truth value F only when both p and q have the truth value F. otherwise True (T).

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional statement: [If .... then] [$\rightarrow$]

Let p and q in any two statements then the statement $p \rightarrow q$ is called conditional statement. (read has if p then q). $p \rightarrow q$ has the truth value F if p has the truth value T. And q has the truth value F. otherwise remaining cases it has the truth value T.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$q \rightarrow p$ ~~$p \leftarrow q$~~
T	T	T
T	F	T
F	T	F
F	F	T

Biconditional: ($\leftrightarrow$) [if and only if] [iff]

Let p and q be any two statements then the statement $p \leftrightarrow q$ which is read as p if and only if q is called biconditional statement. The statement $p \leftrightarrow q$ has the truth value T whenever both p and q have same truth value.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Negation:- [~ (or) ¬] (not)

If p has the truth value T then [~ (or) ¬] ~p has the truth value F. If truth value is F then ~p has the truth value T.

1. Ex:

If Malathi is rich, Malathi is Happy write in Symbolic form

1. Malathi is poor but happy
2. " " rich or unhappy
3. " " neither rich nor happy
4.

$\sim p \wedge q$

$p \vee \sim q$

$\sim p \wedge \sim q$

Sol:

Let p, Malathi is rich: then q will be Malathi is happy.

1. $\sim p \wedge q$

$p \vee \sim q$

1) ~p Malathi is poor
q - Happy
$\sim p \vee q$

2) p - Malathi is rich
~q - Unhappy
$p \vee \sim q$

3) ~p - Malathi is neither rich
~q - nor happy
$\sim p \vee \sim q$

2. How can this english sentence be translate into logical expression.

You can access the internet from campus only. If you are computer science major or you are not a freshman.

Sol:

Let p : you can access the internet from campus only

q : If you are a computer science major.

r : ~~or~~ you are ~~not~~ a freshman.

~r : or you are not a freshman.

∴ The logical expression is $p \rightarrow (q \vee \sim r)$

3) Construct the truth table for $(p \vee q) \vee \sim q$

p	q	$p \vee q$	$\sim q$	$(p \vee q) \vee \sim q$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

∴ It is a tautology.

Find the truth table. $(P \rightarrow q) \leftrightarrow (\sim P \vee q)$

P	q	$\sim P$	$P \rightarrow q$	$\sim P \vee q$	$(P \rightarrow q) \leftrightarrow (\sim P \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$\sim P \wedge (\sim q \wedge r) \vee ((q \wedge r) \vee (P \wedge r))$

Construct truth table for $s: \cancel{(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))}$

P	q	r	$\sim P$	$\sim q$	$q \wedge r$	$\sim q \wedge r$	$P \wedge r$
T	T	T	F	F	T	F	T
T	T	F	F	F	F	F	F
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	F
F	T	T	T	F	T	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	F
F	F	F	T	T	F	F	F

$\sim P \wedge (\sim q \wedge r)$	$(q \wedge r) \vee (P \wedge r)$	$(\sim P \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (P \wedge r))$
F	T	T
F	F	F
F	F	T
F	F	F
F	T	T
F	F	F
T	F	T
F	F	F

Tautology:-

A statement formula which is always true is called tautology.

Contradiction:-

A statement formula which is always false is called contradiction.

Fallacy:-

In the result column any one entry is false is called fallacy.

condigency

A statement formula which is neither tautology nor contradiction is called condigency (both T or F)

show that $(q \vee (P \wedge \sim q)) \vee (\sim P \wedge \sim q)$ is a tautology.

P	q	$\sim q$	$\sim P$	$P \wedge \sim q$	$q \vee (P \wedge \sim q)$	$\sim P \wedge \sim q$	$q \vee (P \wedge \sim q) \vee (\sim P \wedge \sim q)$
T	T	F	F	F	T	F	T
T	F	T	F	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	F	F	T	T

$\therefore$ It is a tautology.

Show that $(P\wedge Q)\wedge \sim(P\vee Q)$ is a contradiction.

P	Q	$P\wedge Q$	$P\vee Q$	$\sim(P\vee Q)$	$(P\wedge Q)\wedge \sim(P\vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore$ The given statement is contradiction.

Check the following proposition is tautology.

$((P\to Q)\to R)\vee \sim P$

P	Q	R	$\sim P$	$P\to Q$	$((P\to Q)\to R)$	$((P\to Q)\to R)\vee \sim P$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

$\therefore$ The given statement is not a tautology.

It is a fallacy.

Logical Equivalences or Equivalence Rules	
Idempotent Laws	$p \wedge p \Leftrightarrow p \quad p \vee p \Leftrightarrow p$
Associative Laws	$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$ $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
Commutative Laws	$p \wedge q \Leftrightarrow q \wedge p$ $p \vee q \Leftrightarrow q \vee p$
De Morgan's Laws	$\neg (p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ $\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
Distributive Laws	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
Complement Laws	$p \wedge \neg p \Leftrightarrow \mathrm{F}$ $p \vee \neg p \Leftrightarrow \mathrm{T}$
Dominance Laws	$p \vee \mathrm{T} \Leftrightarrow \mathrm{T}$ $p \wedge \mathrm{F} \Leftrightarrow \mathrm{F}$
Identity Laws	$p \wedge \mathrm{T} \Leftrightarrow p$ $p \vee \mathrm{F} \Leftrightarrow p$
Absorption Laws	$p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$
Double negation Law	$\neg (\neg p) \Leftrightarrow p$
Contra positive Law	$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$
Conditional as disjunction	$p \rightarrow q \Leftrightarrow \neg p \vee q$
Biconditional as Conditional	$p \rightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$
Exportations Laws	$p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

1. Show that $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

Sol: <u>Given:</u> $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

~~p q ~p ~q p∧~q q∨(p∧~q~~

$= q \vee (p \vee \sim p) \wedge \sim q$ ∴ Distributive law

$= (q \vee p) \vee \sim p \wedge \sim q$ ∴ Associative law

$= (q \vee p) \vee \sim (p \vee q)$ ∴ Complement law

$= T$

∴ It is tautology.

2. Show that $(p \wedge q) \rightarrow (p \vee q)$ is tautology.

$(p \wedge q) \rightarrow (p \vee q)$ ~~Computation~~ conditional as disjunction

$= \sim (p \wedge q) \vee (p \vee q)$

$= (\sim p \vee \sim q) \vee (p \vee q)$ De morgan's

$= (\sim p \vee p) \vee (\sim q \vee q)$ Associative

$= T \vee T$

$= T$

2. Determine whether $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$ is a tautology.

Sol:

Given:-

$(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

$= (\sim q \wedge (\sim p \vee q)) \rightarrow \sim p$ ∴ $p \rightarrow q = \sim p \vee q$

$= \sim (\sim q \wedge (\sim p \vee q)) \vee \sim p$ ∴ $p \rightarrow q = \sim p \vee q$

$= Q \vee \sim(\sim P \vee q) \vee \sim P$ ∴ Demorgan's law

$= Q \vee (P \wedge \sim q) \vee \sim P$ ∴ Demorgan's law

$= (Q \vee P) \wedge (Q \vee \sim q) \vee \sim P$ Distributive law

$= ((Q \vee P) \wedge T) \vee \sim P$ complement

$= (Q \vee P) \vee \sim P$ Identity

$= q \vee (P \vee \sim P)$ Associative

$= q \vee T$ Dominance

$= T$

4. Show that $((P \vee q) \wedge \sim(\sim P \wedge (\sim q \vee \sim r))) \vee (\sim P \wedge \sim q) \vee (\sim P \wedge \sim r)$ is a tautology.

Sol:

Given

$((P \vee q) \wedge \sim(\sim P \wedge (\sim q \vee \sim r))) \vee (\sim P \wedge \sim q) \vee (\sim P \wedge \sim r)$

Consider

$= \sim(\sim P \wedge (\sim q \vee \sim r))$

$= \sim(\sim P \wedge \sim q) \vee (\sim P \wedge \sim r)$ Distributive law

$= (P \vee q) \vee (P \cancel{\wedge} r)$ — ① ∴ Demorgan's law.

Consider

$(\sim P \wedge \sim q) \vee (\sim P \wedge \sim r)$

$\cancel{\sim(P \vee q) \wedge (P \vee r)}$ Demorgan

$\sim(P \vee q) \cancel{\wedge} \sim(P \vee r)$ ∴ Demorgan's law

$\sim((P \vee q) \wedge (P \vee r))$ — ② ∴ Demorgan's law.

$((P \vee q) \wedge ((P \vee q) \vee (P \vee r))) \vee (\sim((P \vee q) \wedge (P \vee r)))$ Idempotent

$= (P \vee q) \wedge (P \vee q) \wedge (P \vee r) \vee (\sim(P \vee q) \wedge (P \vee r))$ complement

$= (P \vee q) \wedge (P \vee r) \vee \sim(P \vee q) \wedge (P \vee r)$

$= T$

5. Show that $(\sim P \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (P \wedge r) \Leftrightarrow r$

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Sol

$(\sim P \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (P \wedge r)$

$\Rightarrow (\sim P \wedge (\sim q \wedge r)) \vee ((q \vee P) \wedge r))$ ∴ Distributive

$\Rightarrow ((\sim P \wedge \sim q) \wedge r)) \vee ((q \vee P) \wedge r)$ ∴ Associative

$\Rightarrow ((\sim P \wedge \sim q) \vee (q \vee P)) \wedge r$ ∴ Distributive

$\Rightarrow (\sim (P \vee q) \vee (q \vee P)) \wedge r$ ∴ Demorgan's

$\Rightarrow T \wedge R$ ∴ $P \vee \sim P \Leftrightarrow T$

$\Rightarrow R$ ∴ $P \wedge T \Leftrightarrow P$

6. Show that $(P \rightarrow (q \rightarrow r)) \rightarrow ((P \rightarrow q) \rightarrow (P \rightarrow r))$ is a tautology.

Sol:

$(P \rightarrow (q \rightarrow r)) \rightarrow (P \rightarrow q) \rightarrow (P \rightarrow r))$

$\Rightarrow (P \rightarrow q) \rightarrow (P \rightarrow r)) \rightarrow ((P \rightarrow q) \rightarrow (P \rightarrow r))$

∴ $P \rightarrow (q \rightarrow r) \Leftrightarrow$
$(P \rightarrow q) \rightarrow (P \rightarrow r)$

$\Rightarrow \sim ((P \rightarrow q) \rightarrow (P \rightarrow r)) \vee ((P \rightarrow q) \rightarrow (P \rightarrow r))$

$\Rightarrow T$

∴ $P \rightarrow q \Leftrightarrow \sim P \vee q$

∴ $\sim P \vee P \Leftrightarrow T$

Equivalents:-

Two statements for laws A & B are equivalent if and only if (iff) $A \leftrightarrow B$ or $A \rightleftarrows B$ is a tautology. It is denoted by the symbol $A \Leftrightarrow B$ which is read as A is equivalent to B.

1. Show that $\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Leftrightarrow (\sim P \vee Q)$

sol:

Given

$\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q))$

$\Rightarrow \sim(\sim(P \wedge Q)) \vee \sim P \vee (\sim P \vee Q)$ ∴ Conditional as disjunction.

$\Rightarrow (P \wedge Q) \vee \sim P \vee (\sim P \vee Q)$ ∴ Double negation.

$\Rightarrow (P \wedge Q) \vee (\sim P \vee \sim P) \vee Q$ ∴ ~~Associative~~ Associative. ~~Idempotent~~

$\Rightarrow (P \wedge Q) \vee (\sim P \vee Q)$ ∴ Idempotent

$\Rightarrow P \vee (\sim P \vee Q) \wedge (Q \vee \sim P \vee Q)$ ∴ Distributive

$\Rightarrow (P \vee \sim P) \vee Q) \wedge (Q \vee Q \vee \sim P)$ ∴ Associative

$\Rightarrow (T \vee Q) \wedge (Q \vee \sim P)$ $P \vee \sim P \Leftrightarrow T$, $Q \vee Q \Leftrightarrow Q$.

$\Rightarrow T \wedge (Q \vee \sim P)$ $T \vee Q \Leftrightarrow T$

$\Rightarrow Q \vee \sim P$ $\Rightarrow P \wedge T \Leftrightarrow P$.

Show that p is equivalent to the following

(i) $p \wedge p$
(ii) $p \vee p$
(iii) $p \vee (p \wedge q)$
(iv) $p \wedge (p \vee q)$ by truth tables.

Sol:

i) $p \Leftrightarrow p \wedge p$

ii)

Show that $\sim(p \vee (\sim p \wedge q))$ and $(\sim p \wedge \sim q)$ are logically equivalent.

Prove that $(p \vee q) \wedge (\sim p \wedge (\sim p \wedge q)) \Leftrightarrow \sim p \wedge q$

Sol:

$(p \vee q) \wedge (\sim p \wedge (\sim p \wedge q))$

$\Leftrightarrow (p \vee q) \wedge ((\sim p \wedge \sim p) \wedge q))$ ∴ Associative

$\Leftrightarrow (p \vee q) \wedge (\sim p \wedge q)$ ∴ Idempotent

$\Leftrightarrow (p \vee q) \wedge (q \wedge \sim p)$ ∴ Commutative

$\Leftrightarrow ((p \vee q) \wedge q) \wedge \sim p$ ∴ Associative

$\Leftrightarrow (q \wedge (q \vee p)) \wedge \sim p$ ∴ Commutative

$\Leftrightarrow q \wedge \sim p$ ∴ Absorption

$\Leftrightarrow \sim p \wedge q$ ∴ Commutative.

Prove that $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

Sol:

$p \rightarrow (q \rightarrow r)$

$\Rightarrow \neg p \vee (q \rightarrow r)$ ∴ $p \rightarrow q \Leftrightarrow \neg p \vee q$ cond & dis cond

$\Rightarrow \neg p \vee (\neg q \vee r)$

$\Rightarrow (\neg p \vee \neg q) \vee r$ ∴ Associative Law

$\Rightarrow \neg (p \wedge q) \vee r$ ∴ De morgan's law

$\Rightarrow (p \wedge q) \rightarrow r$ ∴ $p \rightarrow q \Leftrightarrow \neg p \vee q$

Tautological implication:

A statement formula A is said to be tautologically imply. A statement formula B, iff $A \to B$ is a tautology. We shall denote this idea by $A \Rightarrow B$ which is read as $A \to B$

Show that $(P \wedge Q) \Rightarrow (P \to Q)$ by truth table.

Soln

P	Q	$P \wedge Q$	$P \to Q$	$(P \wedge Q) \to (P \to Q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

It is enough to prove $P \wedge Q \to (P \to Q)$

∴ It is a tautology.

Prove that $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$ by using truth table.

P	Q	R	$Q \to R$	$P \to Q$	$P \to R$	$P \to (Q \to R)$	$(P \to Q) \to (P \to R)$	$(P \to (Q \to R)) \to (P \to Q) \to (P \to R)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Show that $[(p\to q)\to q] \Rightarrow (p\vee q)$ without truth table

Sol: It is enough to prove that $[(p\to q)\to q]\to(p\vee q)$ is a tautology.

Consider

$[(p\to q)\to q]\to[p\vee q]$

$\Leftrightarrow [(\sim p\vee q)\to q]\to(p\vee q)$ — $p\to q\Leftrightarrow \sim p\vee q$

$\Leftrightarrow [\sim(\sim p\vee q)\vee q]\to(p\vee q)$ — "

$\Leftrightarrow [(p\wedge\sim q)\vee q]\to(p\vee q)$ — "

$\Leftrightarrow \sim[(p\wedge\sim q)\vee q]\vee(p\vee q)$ — "

$\Leftrightarrow [(\sim p\vee q)\wedge\sim q]\vee(p\vee q]$ — Distributive

$\Leftrightarrow [(\sim p\wedge\sim q)\vee(q\wedge\sim q)]\vee[p\vee q]$

$\Leftrightarrow [(\sim p\wedge\sim q)\vee F]\vee[p\vee q]$ — $q\wedge\sim q\Leftrightarrow F$

$\Leftrightarrow [(\sim p\wedge\sim q)]\vee[p\vee q]$ — $p\vee F\Leftrightarrow p$

$\Leftrightarrow \sim(p\vee q)\vee(p\vee q)$ — Demorgan law

$\Leftrightarrow T$

Prove that $(p\to q)\Rightarrow p\to(p\wedge q)$

Sol: Enough to prove

$(p\to q)\to p\to(p\wedge q)$ is a tautology.

$\Leftrightarrow \sim(\sim p\vee q)\vee(\sim p\vee(p\vee q))$ — $p\to q\Leftrightarrow\sim p\vee q$

$\Leftrightarrow (p\wedge\sim q)\vee(\sim p\vee(p\wedge q))$ — Demorgan's

$\Leftrightarrow (p\wedge\sim q)\vee((\sim p\vee p)\wedge(\sim p\vee q))$ — Distributive

$\Leftrightarrow (p\wedge\sim q)\vee(T\wedge(\sim p\vee q))$ — Complement

$\Leftrightarrow (p\wedge\sim q)\vee(\sim p\vee q)$ — Identity

$\Leftrightarrow (p\vee(\sim p\vee q))\wedge(\sim q\vee(\sim p\vee q))$ — Distributive

$\Leftrightarrow ((p\vee\sim p)\vee q)\wedge((\sim q\vee q)\vee\sim p)$ — Associative

$\Leftrightarrow (T\vee q)\wedge(T\vee\sim p)$ — Complement

$\Leftrightarrow T\wedge T$ — ~~Identity~~ Dominance

$\Leftrightarrow T$

Converse, Contrapositive, Inverse :-

Let $P \to q$ be a preposition then

the converse of $P \to q$ is $q \to P$

The Contrapositive of $P \to q$ is $\sim q \to \sim P$

The inverse of $P \to q$ is $\sim P \to \sim q$

1. Give the Converse and contrapositive of the implication.

"If it is raining, then I get wet".

Sol:

Let P : It is raining

q : I get wet

Given:

$P \to q$

$\therefore$ The Converse of $P \to q$ is $q \to P$

i.e.,

I get wet If it is raining.

$\therefore$ The contrapositive of $P \to q$ is $\sim q \to \sim P$

i.e.,

If I do not get wet then it is not raining.

Dual of a statement formula:-

Two formula A and A* are said to be dual of each other if either one can be obtained from other by replacing $\wedge$ by $\vee$, $\vee$ by $\wedge$, T by F, F by T.

1. Dual of $(P \wedge Q) \vee T$ is $(P \vee Q) \wedge F$

2. $P \wedge \sim(Q \vee \sim S) = P \vee \sim(Q \wedge \sim S)$

Normal forms:-

The Convenient to use the word "product" inplace of conjuction and "sum" inplace of disjunction.

Elementary Sum (OR) ($\vee$)

Disjunction of the statement Variable and the negations is known as elementary sum.

Ex:

$P \vee Q$, $\sim P \vee Q$, $P \vee \sim Q$, $\sim P \vee \sim Q$.

Elementry product: (AND) ($\wedge$)

The conjuction of the statement variables and their negations is said to be elementry product.

Ex:

$p\wedge q$, $p\wedge\sim q$, $\sim p\wedge q$, $\sim p\wedge\sim q$

Types of normal form:-

1. Disjuctive normal form (DNF)
2. Conjunctive normal form (CNF)

1. DNF

A statement formula which is equivalent to a given formula which can be expressed as the disjunction of elementry product. is known as DNF.

Ex:

Distributive law

$$p\wedge(q\vee r) = \cancel{p\vee(q\wedge r)}\,(p\wedge q)\vee(p\wedge r)$$

CNF:-

A statement formula which is equivalent to a given formula which can be expressed as the conjunction of elementry sum is known as CNF.

Ex:

$$p\vee(q\wedge r) = (p\vee q)\wedge(p\vee r)$$ ∴ Distributive law.

DNF :- (Elemetry product) $\vee$ (Elementry product) $\vee \ldots \vee$ (Elementry product)

CNF :- (Elementry sum) $\vee$ (Elementry sum) $\vee \ldots \vee$ (Elementry sum)

Min terms:-

Let $P \wedge q$ with two statement variables. Let us construct all possible formulas which consists of conjuction of P or its negation and conjuction of q or its negation.

None of the formula should contain both a variable and its negation.

Using commutative law if any two terms are equivalent choose any one of the term. Collect the remaining terms they are called min terms.

Ex:

$P \wedge q, P \wedge \sim q, \sim P \wedge q, \sim P \wedge \sim q$

Result:-

1. $P \wedge q$ or $q \wedge P$ is included but not both.
2. $P \wedge \sim P$ and $q \wedge \sim q$ are not allowed.
3. No two min terms are equivalent.
4. Min terms of 3 variables P, q, r

$P \wedge q \wedge r, \sim P \wedge q \wedge r, P \wedge \sim q \wedge r, P \wedge q \wedge \sim r,$
$\sim P \wedge \sim q \wedge r, \sim P \wedge q \wedge \sim r, P \wedge \sim q \wedge \sim r, \sim P \wedge \sim q \wedge \sim r.$

5. In general for given n number of variable there will be 2^n min terms.

~~a/ Biconditional equivalence~~

r.

6. $P \rightarrow q \Leftrightarrow \sim P \vee q$

$P \rightarrow q \Leftrightarrow \sim q \rightarrow \sim P$

7. Biconditional equivalence:

$$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$$
$$P \leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$$
$$P \leftrightarrow Q \Leftrightarrow (\sim P \vee Q) \wedge (\sim Q \vee P)$$

Max terms:

For a given number of variables the max term consists of disjunction in which each variable or its negation but not both appears only once.

Ex:

$P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$, $\sim P \vee \sim Q$.

Principle disjunctive normal form: (PDNF)

For a given statement formula and equivalent formula consisting of disjunction of min terms only is known as PDNF or sum of product of canonical form.

i.e.,

$$PDNF = \begin{pmatrix} \text{min} \\ \text{terms} \end{pmatrix} \vee \begin{pmatrix} \text{min} \\ \text{term} \end{pmatrix} \vee \dots \vee \begin{pmatrix} \text{min} \\ \text{terms} \end{pmatrix}$$

Principle Conjunctive normal form: (PCNF)

For a given statement formula and equivalent formula consisting of conjuction of max terms only is known as PCNF (or) product of sum of canonical form.

(i.e.),

$$PCNF = \begin{pmatrix} \text{max} \\ \text{term} \end{pmatrix} \vee \begin{pmatrix} \text{max} \\ \text{terms} \end{pmatrix} \vee \dots \vee \begin{pmatrix} \text{max} \\ \text{terms} \end{pmatrix}$$

1. Obtain PDNF of $\sim p \vee q$ (or) $p \to q$ also find PCNF.

Sol:

p	q	$\sim p$	$\sim p \vee q$	min term	max term
T	T	F	T	$p \wedge q$	--
T	F	F	F	-	$\sim p \vee q$
F	T	T	T	$\sim p \wedge q$	-
F	F	T	T	$\sim p \wedge \sim q$	-

PDNF is

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

PCNF is

$(\sim p \vee q)$

2. Obtain PDNF & PCNF for $(\sim P \to R) \wedge (q \leftrightarrow P)$

p	q	r	$\sim p$	$\sim p \to r$	$q \leftrightarrow p$	$(\sim p \to r) \wedge (q \leftrightarrow p)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

$p \wedge q \wedge r$

$\sim p \vee q \vee \sim r$

$\sim p \wedge q \wedge \sim r$

$p \vee q \vee r$

Min terms	max terms.
$P \wedge q \wedge r$	–
$P \wedge q \wedge \sim r$	–
–	$\sim P \vee q \vee \sim r$
–	$\sim P \vee q \vee r$
–	$P \vee \sim q \vee \sim r$
–	$P \vee \sim q \vee r$. kya
$\sim P \wedge \sim q \wedge r$	–
–	$P \vee q \vee r$

PDNF is

$(P \wedge q \wedge r) \vee (P \wedge q \wedge \sim r) \vee (\sim P \wedge \sim q \wedge r)$

PCNF is

$(\sim P \vee q \vee \sim r) \wedge (\sim P \vee q \vee r) \wedge (P \vee \sim q \vee \sim r) \wedge (P \vee \sim q \vee r) \wedge (P \vee q \vee r)$

Working rule to obtain PDNF.

1. To replace conditional and biconditional by their equivalence formula involving $\wedge, \vee, \& \sim$ only.
2. Apply the fact (each term) $\wedge T$ ($\because P \wedge T \Leftrightarrow P$)
3. Apply distributive rule.
4. Apply Commutative rule.

Working rule to obtain PCNF:

1. To replace conditional & biconditional by their equivalence formula involving $\wedge, \vee, \sim$.
2. Apply the fact (each term) $\vee F$. ($\because P \vee F \Leftrightarrow P$)
3. Apply distributive law
4. Apply Commutative law.

1\. Find PDNF of $(P\wedge q)\vee(\sim P\wedge r)\vee(q\wedge r)$

sol:

$(P\wedge q)\vee(\sim P\wedge r)\vee(q\wedge r)$

$\Leftrightarrow (P\wedge q\wedge T)\vee(\sim P\wedge r\wedge T)\vee(q\wedge r\wedge T)$ $\quad \therefore P\wedge T \Leftrightarrow P$

$\Leftrightarrow (P\wedge q\wedge(R\vee\sim r))\vee(\sim P\wedge r\wedge(q\vee\sim q))\vee(q\wedge r\wedge(P\vee\sim P))$

$\Leftrightarrow (P\wedge q\wedge r)\vee(P\wedge q\wedge\sim r)\vee(\sim P\wedge r\wedge q)\vee(\sim P\wedge r\wedge\sim q)$
$\vee(q\wedge r\wedge P)\vee(q\wedge r\wedge\sim P)$

$\Leftrightarrow (P\wedge q\wedge r)\vee(P\wedge q\wedge\sim r)\vee(\sim P\wedge q\wedge r)\vee(\sim P\wedge\sim q\wedge r)$
$\vee(P\wedge q\wedge r)\vee(\sim P\wedge q\wedge r)$

The PDNF is,

$(P\wedge q\wedge r)\vee(P\wedge q\wedge\sim r)\vee(\sim P\wedge q\wedge r)\vee(\sim P\wedge\sim q\wedge r)$.

2\. Obtain PCNF of the formula $S:(\sim P\rightarrow r)\wedge(q\Leftrightarrow P)$ and hence obtain PDNF (or) Find product of sum canonical form of the formula $S:(\sim P\rightarrow r)\wedge(q\Leftrightarrow P)$ and hence obtain sum of product canonical form.

sol:

Given:

$S:(\sim P\rightarrow r)\wedge(q\Leftrightarrow P)$

$\Leftrightarrow (\sim(\sim P)\vee r)\wedge(q\rightarrow P)\wedge(P\rightarrow q)$

$\Leftrightarrow (P\vee r)\wedge(\sim q\vee P)\wedge(\sim P\vee q)$

$\Leftrightarrow (P\vee r\vee F)\wedge(\sim q\vee P\vee F)\wedge(\sim P\vee q\vee F)$

$\Leftrightarrow (P\vee r\vee(q\wedge\sim q))\wedge(\sim q\vee P\vee(r\wedge\sim r))\wedge(\sim P\vee q\vee(r\wedge\sim r))$

$\Leftrightarrow (P\vee r\vee q)\wedge(P\vee r\vee\sim q)\wedge(\sim q\vee P\vee r)\wedge(\sim q\vee P\vee\sim r)$
$\wedge(\sim P\vee q\vee r)\wedge(\sim P\vee q\vee\sim r)$

$\Leftrightarrow (P\vee q\vee r)\wedge(P\vee\sim q\vee r)\wedge(P\vee\sim q\vee r)\wedge(P\vee\sim q\vee\sim r)$
$\wedge(\sim P\vee q\vee r)\wedge(\sim P\vee q\vee\sim r)$

The PCNF is

$(p\vee q\vee r)\wedge(p\vee\sim q\vee r)\wedge(p\vee\sim q\vee\sim r)\wedge(\sim p\vee q\vee r)$

$\wedge(\sim p\vee q\vee\sim r)$

Let $\sim S$:-

$(p\vee q\vee\sim r)\wedge(\sim p\vee\sim q\vee r)\wedge(\sim p\vee\sim q\vee\sim r)$

($\because$ Remaining maxterm)

The PDNF is $\sim(\sim S)$

$=\sim[(p\vee q\vee\sim r)\wedge(\sim p\vee\sim q\vee r)\wedge(\sim p\vee\sim q\vee\sim r)]$

$=(\sim p\wedge\sim q\wedge r)\vee(p\wedge q\wedge\sim r)\vee(p\wedge q\wedge r)$

3) Find PDNF of $(p\wedge q)\vee(\sim p\wedge r)$ also find PCNF:-

Sol:

Given

$(p\wedge q)\vee(\sim p\wedge r)$

$\Leftrightarrow(p\wedge q\wedge T)\vee(\sim p\wedge r\wedge T)$

$\Leftrightarrow(p\wedge q\wedge(r\vee\sim r))\vee(\sim p\wedge r\wedge(q\vee\sim q))$

$\Leftrightarrow(p\wedge q\wedge r)\vee(p\wedge q\wedge\sim r)\vee(\sim p\wedge r\wedge q)\vee(\sim p\wedge r\wedge\sim q)$

$\Leftrightarrow(p\wedge q\wedge r)\vee(p\wedge q\wedge\sim r)\vee(\sim p\wedge q\wedge r)\vee(\sim p\wedge\sim q\wedge r)$

$\therefore$ PDNF is.

$(p\wedge q\wedge r)\vee(p\wedge q\wedge\sim r)\vee(\sim p\wedge q\wedge r)\vee(\sim p\wedge\sim q\wedge r)$

Let $\sim S$ is

$(p\wedge\sim q\wedge r)\vee(p\wedge\sim q\wedge\sim r)\vee(\sim p\wedge q\wedge\sim r)\vee(\sim p\wedge\sim q\wedge\sim r)$

$\therefore$ PCNF is

$\sim(\sim S)$

$=\sim[(p\wedge\sim q\wedge r)\vee(p\wedge\sim q\wedge\sim r)\vee(\sim p\wedge q\wedge\sim r)\vee(\sim p\wedge\sim q\wedge\sim r)]$

$=(\sim p\vee q\vee\sim r)\wedge(\sim p\vee q\vee r)\wedge(p\vee\sim q\vee r)\wedge(p\vee q\vee r)]$

Find PCNF & PDNF for $(p \to (q \wedge r)) \wedge (\sim p \to (\sim q \wedge \sim r))$

sol:

Given

$(p \to (q \wedge r)) \wedge (\sim p \to (\sim q \wedge \sim r))$

$= (\cancel{p \to q \wedge r})$

$= (\sim p \vee q \wedge r) \wedge [\sim(\sim p \vee (\sim q \wedge \sim r))]$

$= \sim(\sim p \vee q \wedge r) \wedge [\sim p \vee (\sim q \wedge \sim r))]$

$= (\sim p \vee q) \wedge (\sim p \vee r) \wedge (p \vee \sim q) \wedge (p \vee \sim r)$

$= (\sim p \vee q \vee F) \wedge (\sim p \vee r \vee F) \wedge (p \vee \sim q \vee F) \wedge (p \vee \sim r \vee F)$

$= (\sim p \vee q \vee (r \wedge \sim r)) \wedge (\sim p \vee r \vee (q \wedge \sim q)) \wedge (p \vee \sim q \vee (r \wedge \sim r)) \wedge (p \vee \sim r \vee (q \wedge \sim q))$

$= (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee r \vee q) \wedge (\sim p \vee r \vee \sim q) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee \sim r \vee q) \wedge (p \vee \sim r \vee \sim q)$

$= (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r)$

ie PCNF is,

$= (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee q \vee \sim r)$

Let $\sim S$ is

$$(\sim p \vee \sim q \vee \sim r) \wedge (p \vee q \vee r)$$

The PDNF is,

$\sim(\sim S)$ is

$$= \sim[(\sim p \vee \sim q \vee \sim r) \wedge (p \vee q \vee r)]$$

$$= (p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$$

Find the PCNF & PDNF for $(p \wedge q) \vee (\sim p \wedge r)$.

The theory of inference

Premises (or) hypothesis:-

Premises is a statement which is assumed to be true.

Rule of inference:- 2m

A set of premises $H_1, H_2 \ldots H_n$ and a conclusion C are given. We assume that $H_1, H_2 \ldots H_n$ are all true. We want to conclude the conclusion C is proved. The following tools are used to two rules,

1. **Rule P:-**

A premises may be introduced at any point in the derivation

2. **Rule T:-**

A formula S may be introduced in a derivation if S is tautologically implied by anyone or more of the preceding formulas in the derivation.

Types of proof:-

1. Direct proof
2. Indirect "
3. Conditional conclusion.

Implication rules:-

1. $P, P \to q \Rightarrow q$ (modus phones)
2. $\sim q, P \to q \Rightarrow \sim P$ (Modus tollens)
3. $\sim P, P \vee q \Rightarrow q$ (Disjunctive syllogism)
4. $P \to q, q \to r \Rightarrow P \to r$ (hypothetical syllogism (or) chain rule).

5. $P, q \Rightarrow P \wedge q$ } simplication rule.
6. $P \wedge q \Rightarrow P, q$ }
7. $P, q \Rightarrow P \vee q$ (addition rule)
8. $P \wedge \sim q \Leftrightarrow \sim(P \to q)$ (Equivalence rule).

Demonstrate that R is a valid inference from the premises $P \to q$, $q \to r$ and P.

Sol:

Given premises

$$P \to q, \ q \to r \ \& \ P$$

Conclusion is R.

steps	premises	justification.
1.	$P \to q$	Rule P
2.	$q \to r$	Rule P
3.	$P \to r$	Rule T ($(P \to q), q \to r \Rightarrow P \to r$)
4.	P	Rule P
5.	R	Rule T ($P, P \to q = q$)

Alter:-

steps	premises	justification
1.	p	Rule P
2.	$p \to q$	Rule P
3.	q	Rule T $(p, p \to q \Rightarrow q)$
4.	$q \to r$	Rule P
5.	R	Rule T $(q, q \to r \Rightarrow r)$

Show that $R \vee S$ follows logically from the premises $c \vee d$, $(c \vee d) \to \sim h$, $\sim h \to (a \wedge \sim b)$ and $(a \wedge \sim b) \to R \vee S$

Sol:

Given

$c \vee d$, $(c \vee d) \to \sim h$, $\sim h \to (a \wedge \sim b)$ and $(a \wedge \sim b) \to R \vee S$

Conclusion is $R \vee S$

$C \vee D$	Rule P
$C \vee D$, $(C \vee D) \to \sim H$	Rule P
$\sim H$	Rule T
$\sim H \to (A \wedge \sim B)$	Rule P
$A \wedge \sim B$	Rule T
$(A \wedge \sim B) \to R \vee S$	Rule P
$R \vee S$	Rule T.

Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\sim M$.

Sol:

Given conclusion $R \wedge (P \vee Q)$

$P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$, $\sim M$

Step	Premises	Justification.
1.	$P \rightarrow M$	P
2.	$\sim M$	P.
3.	$\sim P$	T $\sim P, P \rightarrow Q \Rightarrow \sim P$
4.	$P \vee Q$	P
5.	Q	T
6.	$Q \rightarrow R$	P
7.	R	T
8.	$R \wedge (P \vee Q)$	T $R, Q \Rightarrow P \wedge Q$ (I9)

Show that $R \rightarrow \sim Q$, $R \vee S$, $S \rightarrow \sim Q$, $P \rightarrow Q \Rightarrow \sim P$.

$R \rightarrow \sim Q$

$S \rightarrow \sim Q$

$R \rightarrow \sim Q$

$\sim P \vee \sim Q$

$R \vee S$

$S \rightarrow \sim Q$

$\sim S \vee \sim Q$

Rule CP :-

If the derived s from R and a set of premises than we can derive $r \rightarrow s$. from the set of premises alone.

show that $R \rightarrow S$ can be derived from the Premises $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ and Q.

Sol:

Given:- $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ & Q.

conclusion $R \rightarrow S$

step	premises	justification.
1.	$\sim R \vee P$	P
2.	$R \rightarrow P$	T $P \rightarrow Q \Leftrightarrow \sim P \vee Q$
3.	R	Additional premise
4.	P	T $P, P \rightarrow Q \Rightarrow Q$
5.	$P \rightarrow (Q \rightarrow S)$	P
6.	$Q \rightarrow S$	T
7.	Q	P
8.	S	T
9.	$R \rightarrow S$	Rule CP.

5. Show that $P \to (Q \to S)$ can be derived from a set of premises $P \to (Q \to R)$, $Q \to (R \to S)$

1.	P	Assume premise
2.	$P \to (Q \to R)$	P.
3.	$Q \to R$	T
4.	$Q \to (R \to S)$	P
5.	$\sim Q \vee (\sim R \vee S)$	T
6.	$\sim R \vee (\sim Q \vee S)$	T Associative law
7.	$R \to (Q \to S)$	T
8.	$Q \to (Q \to S)$	T (3, 7)
9.	$\sim Q \vee (\sim Q \vee S)$	T
10	$(\sim Q \vee \sim Q) \vee S$	T Associative
11	$\sim Q \vee S$	T Idempedent
12	$Q \to S$	T
13.	$P \to (Q \to S)$	Rule CP

Derive the following (use CP rule if necessary)

$\sim P \vee q, \sim q \vee r, r \rightarrow s \Rightarrow P \rightarrow s$

Sol:

Given:-

$\sim P \vee q, \sim q \vee r, r \rightarrow s$

Conclusion is $P \rightarrow s$

1.	$\sim P \vee q$	P
2.	$P \rightarrow q$	T
3.	P	Additional premise
4.	Q	T (2,3)
5.	$\sim Q \vee R$	P
6.	$Q \rightarrow R$	T
7.	R	T (4,6)
8.	$R \rightarrow S$	P
9.	S	T (7,8)
10	$P \rightarrow S$	Rule CP.

Inconsistency of premises:-

A set of premises $H_1, H_2 \ldots H_n$ is said to be inconsistent if the conjunction implies contradiction. i.e., $H_1 \wedge H_2 \wedge \ldots \wedge H_n \Rightarrow R \wedge \sim R$ where R is any formula (note that $R \wedge \sim R \Rightarrow F$)

Show that $P \to q$, $P \to r$, $q \to \sim r$ and P are inconsistent.

sol:

Given:

$P \to q$, $P \to r$, $q \to \sim r$ & P

Steps	premises	justification.
1.	P	Rule P
2.	$P \to q$	Rule P
3.	q	Rule T
4.	$q \to \sim r$	Rule P
5.	$\sim r$	Rule T
6.	$P \to r$	Rule P
7.	$\sim P$	Rule T.
(1,7) 8	$P \vee \sim P$	T

$\therefore$ Given premises are Inconsistent.

Prove that $P\to q$, $q\to r$, $S\to\sim r$, $P\wedge S$ are inconsistent.

Sol:

Given

$P\to q$, $q\to r$, $S\to\sim r$, $P\wedge S$

steps	premises	justification.
1.	$P\to q$	P
2.	$q\to r$	P
3.	$P\to r$	T $(P\to q, q\to r \Rightarrow P\to r)$
4.	$S\to\sim r$	P
5	$R\to\sim S$	T $(P\to q \Rightarrow \sim q\to\sim P)$
6.	$P\to\sim S$	T (3, 5)
7.	$\sim P\vee\sim S$	T $(P\to q \Rightarrow \sim P\vee q)$
8.	$\sim(P\wedge S)$	T (De Morgan's)
9.	$P\wedge S$	P
10	$(P\wedge S)\wedge\sim(P\wedge S)$	T $(P, q \Rightarrow P\wedge q)$
11	F	T

Indirect method of proof

To show that the conclusion C follows logically from the premises $H_1, H_2, \ldots H_n$, we assume C is false and consider $\sim C$ as an additional premises. If $H_1 \wedge H_2 \wedge \ldots \wedge H_n \wedge \sim C$ is a Contradiction. Then, C follows logically from $H_1, H_2, \ldots H_n$.

Using Indirect method show that $p \rightarrow q, q \rightarrow r, p \vee r \Rightarrow r$.

Sol.

Given

$p \rightarrow q, q \rightarrow r, p \vee r \Rightarrow r$

We introduce $\sim r$ as an additional premise and show that this additional premise lead to a contradiction.

Steps	premises	justification,
1.	$\sim r$	Assumed premise
2.	$q \rightarrow r$	P
3.	$\sim q$	T
4.	$p \rightarrow q$	P
5.	$\sim p$	T
6.	$p \vee r$	P
7.	$\sim p \rightarrow r$	T

8 8 ~~26~~ r $T(S, \not T)$

$T(P \wedge Q \Rightarrow P \wedge Q)$

9. $R \wedge \sim r$

which is nothing but false therefore by the method of contradiction $(P \to q) \wedge (q \to r) \wedge (P \vee r) \Rightarrow r$.

Show that the hypothesis "It is not sunny this afternoon and it is colder than yesterday". "We will go swimming only if it is sunny." "If we do not go swimming, then we will take a canoe trip." and "If we take a canoe trip then we will be home by sunset." lead to conclusion "we will be home by sunset".

Sol:

Let P : It is sunny this afternoon.
q : It is colder than yesterday.
r : we will go swimming.
s : ~~we~~ will take a canoe trip.
T : we will be home by sunset.

$\sim P \wedge q,\ r \to P,\ \sim r \to s,\ S \to T$

1.	$\sim P \wedge q$	P
		$T\ (P \wedge q \Rightarrow P)$ conjunction simplification.
2.	$\sim P$	
3.	$R \to P$	P
4.	$\sim R$	T
5.	$\sim r \to s$	P
6.	S	T
7.	$S \to T$	P

8.

∴ The given argument is Valid

Show that the following premises are inconsistent.

1) If RajKumar Misses many classes through illness then he fails high school.

If RajKumar fails high school then he is uneducated.

If RajKumar read a lot of books then he is not uneducated.

RajKumar Misses many classes through illness and then read a lot of books.

Sol:

Let

p : RajKumar misses many classes

q : RajKumar fails high school

r : RajKumar read lot of books

s : RajKumar is uneducated.

The premises are $p \to q$, $q \to s$, $r \to \sim s$, $p \wedge r$.

Using Indirect method of proof show that
$P \to R, Q \to S, P \vee Q \Rightarrow S \vee R$.

Sol:

we consider: $\sim(S \vee R)$ as an additional premises.

Steps	Premises	Justification
1.	$\sim(S \vee R)$	Additional premises
2.	$\sim S \wedge \sim R$	T
3.	$\sim S$	T $P \wedge Q \Rightarrow P$
4.	$\sim R$	T $P \wedge Q \Rightarrow Q$
5.	$P \to R$	P
6.	$\sim P$	$T(\sim Q, P \to Q \Rightarrow \sim P)$
7.	$Q \to S$	P
8.	$\sim Q$	T (3,7)
9.	$\sim P \wedge \sim Q$	T (6,8) $P, Q \Rightarrow P \wedge Q$
10	$\sim(P \vee Q)$	T
11.	$P \vee Q$	P
12.	$(P \vee Q) \wedge \sim(P \vee Q)$	$T(P, Q = P \wedge Q)$

which is nothing but false value therefore by the method of contradiction $P \to R$, $Q \to S$, $P \vee Q \Rightarrow S \vee R$.

PREDICATE:-CALCULUS:

The logic based upon the analysis of predicates in any statement is called predicate calculus.

For Ex:-

Consider the sentence "$\underbrace{\text{Ram}}_{\text{subject}}$ $\underbrace{\text{is a boy}}_{\text{predicate}}$"

Here predicate :- is a boy

Subject :- Ram.

which can be symbolise by predicate subject

symbol :- $B(r)$

suppose we can have relation among two object:

Ex:-

Raju is taller than Kannan.

(i.e.) In symbol

$T(r, K)$

suppose we have n object than it is known as enplace predicate.

Statement function and variable:-

A simple statement function of 1 variable is defined to be an expression consisting of a Predicate symbol and an individual variable.

For Ex:-

$M(x)$

x is a man

Such a statement function becomes a statement in the variable is replaced by name of any object.

We can also combine one or more statement function using logical connectivity which from compound statement function.

For ex:-

M(x)

x is a man

H(x) : x is mortal

Then $M(x) \wedge H(x)$, $\rightarrow$, $\sim M(x)$ ...

are examples of compound statement function.

Quantifiers:-

We can indicate the quantity by means of words like "All", "sum" such a word in statement that indicate quantity are called quantifiers.

There are two types of quantifiers,

1. Universal quantifier (All)
2. Existential " (sum)

Universal quantifiers:-

A quantifiers "For all x" is called universal quantifiers.

It is denoted by the symbol $(\forall x)$ (or) (x).

The universal quantifier is equivalent to each of the following phrases.

1. For all x
2. For every x
3. For each x
4. Everything x is such that.
5. Each thing x is "

For ex:-

All cats are animal

sol:

$c(x)$:- x is a cat

$A(x)$:- x is a animal.

In symbol,

$(\forall x)\,(c(x) \longrightarrow A(x))$

Existential Qualifiers:-

The qualifier "for some x" is called existential "

It is denoted by the symbol $(\exists x)$.

The existential quantifier is also equivalent to each of the following phrases,

1. For some x
2. Some x such that
3. There exist an x such that
4. There is an x such that
5. There is atleast one x such that

For ex:

Some men are clever.

Let $M(x)$:- x is a man

$C(x)$: x is a clever.

Symbol:

$(\exists x)[M(x) \wedge C(x)]$

Free & bound Variables:-

1. Variables is said to be bound if it is concerned with either universal ($\forall x$), or Existential ($\exists x$)

2. Variable which is not concerned with any quantifier is called free variable.

Universe of discourse:-

Variables which are quantified stands for only those object which are members of a particular set or class.

Such a set is called universe of discourse.

Symbolic representation of expressions using quantifiers.

1. Write the symbolic form the statement.
"All lions are dangerous".

Sol:
$L(x) = x$ - lion
$d(x) = x$ is dangerous.

$$\forall x, [L(x) \rightarrow D(x)]$$

2. "Some animals are dangerous"

$A(x)$ = x is Animals.

$D(x)$ = x is a dangerous.

Symbol :- $(\exists x)[A(x) \wedge D(x)]$

2. Write the symbolic form "All men are giants".

Sol:

$M(x)$: x is a men

$G(x)$: x is a giants

Symbol :- $(\forall x)[M(x) \rightarrow G(x)]$

Every parrot is ugly.

$P(x)$ = x is a parrot

$U(x)$ = x is ugly.

Symbol : $(\forall x)[P(x) \rightarrow U(x)]$

Express the following statement involving predicating symbol.

1. All students are clever.
2. Some students are not successful.
3. Every clever student successful.

Sol:

1. $S(x)$ = x is a student

$C(x)$ = x is a clever.

Symbol : $(\forall x)[S(x) \rightarrow C(x)]$

2. Some students are not Successful.

$S(x)$ = x is a students

$H(x)$ = x is a successful.

$\sim H(x)$ = x is not a successful.

Symbol :- $(\exists x)[S(x) \wedge \sim H(x)]$

3. Every clever student are Successful.

$S(x)$ = x is a clever student

$H(x)$ = x is a successful.

Symbol :- $(\forall x)[S(x) \rightarrow H(x)]$

1. Universal specification (Rule US) $(\forall x) A(x) \Rightarrow A(y)$
2. Universal Generalisation (Rule UG)

$$A(x) \Rightarrow (\forall y) A(y)$$

3. Existential Specification (Rule ES)

$$(\exists x) A(x) \Rightarrow A(y)$$

4. Existential Generalisation (Rule EG)

$$A(x) \Rightarrow (\exists y) A(y)$$

Some Valid formula :-

1) $(\exists x)[P(x) \wedge Q(x)] \Leftrightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$
2) $(\exists x)[P(x) \vee Q(x)] \Leftrightarrow (\exists x) P(x) \vee (\exists x) Q(x)$
3) $(\exists x)[P(x) \wedge Q(x)] \Leftrightarrow (\forall x) P(x) \wedge (\forall x) Q(x)$
4) $(\forall x)[P(x)] \vee (\forall x)(Q(x)) \Leftrightarrow (\forall x)[P(x) \vee Q(x)]$
5) $\sim[(\forall x) P(x)] \Leftrightarrow (\exists x) \sim P(x)$

6) $\sim[(\exists x)\, P(x)] \Leftrightarrow (\forall x) \sim P(x)$

7) $(\forall x)[P(x) \rightarrow Q(x)] \Leftrightarrow (\forall x)P(x) \rightarrow (\forall x)Q(x)$

8) $(\exists x)(\forall y)P(x,y)$

It's negation is $(\forall x)(\exists y) \sim P(x,y)$

show that $(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)[Q(x) \rightarrow R(x)] \Leftrightarrow (\forall x)[P(x) \rightarrow R(x)]$

Sol:

$P \rightarrow Q,\ Q \rightarrow R \Rightarrow P \rightarrow R$

$P, Q \Rightarrow P \wedge Q$

$P \wedge Q \Rightarrow P$

$P \wedge Q \Rightarrow Q$

Given

$$(\forall x)[P(x) \rightarrow Q(x)] \wedge (\forall x)[Q(x) \rightarrow R(x)] \Leftrightarrow (\forall x)[P(x) \rightarrow R(x)]$$

Steps	premises	justification.
1.	$(\forall x)[P(x) \rightarrow Q(x)]$	P
2.	$P(y) \rightarrow Q(y)$	Rule US
3.	$(\forall x)[Q(x) \rightarrow R(x)]$	P
4.	$Q(y) \rightarrow R(y)$	Rule US
5.	$P(y) \rightarrow R(y)$	T $P \rightarrow Q,\ Q \rightarrow R \Rightarrow P \rightarrow R$
6.	$(\forall x)[P(x) \rightarrow R(x)]$	Rule UG.

$(\forall x)\, A(x) \Rightarrow A(y)$

$A(x) \Rightarrow (\forall y) A(y)$

Show that $\sim P(a,b)$ follows logically from $(x)(y)[P(x,y) \rightarrow W(x,y)]$ and $\sim W(a,b)$

Sol:

Steps.	premises	justification.
1.	$(x)(y)[P(x,y) \rightarrow W(x,y)]$	P
2.	$(y)[P(a,y) \rightarrow W(a,y)]$	Rule US
3.	$P(a,b) \rightarrow W(a,b)$	Rule US
4.	$\sim W(a,b)$	P
5.	$\sim P(a,b)$	T

Show that the following argument is valid.

All rock music is loud music. Some rock music exists. Therefore some loud music exists.

Sol:

$R(x)$:- x is a rock music

$L(x)$:- x is a loud music

Premises:

$(\forall x)[R(x) \rightarrow L(x)]$, $(\exists x) R(x) \Rightarrow (\exists x) L(x)$

Conclusion

Conclusion:- $\exists x\, L(x)$.

$\exists x (L(x))$

Steps	premises	justification.
1.	$\forall x\,[R(x) \to L(x)]$	P
2.	$R(y) \to L(y)$	Rule US.
3.	$(\exists x)\, R(x)$	P
4.	$R(y)$	Rule ES
5.	$L(y)$	T (2, 4)
6.	$(\exists x)\, L(x)$	Rule EG.

show that the premises, A student in this class has not read the book and everyone in this class passed the first exam. imply the conclusion someone who passed the first exam has not read the book.

Sol:

$C(x)$: x is in this class.

$R(x)$: x is read the book.

$P(x)$: x is passed the first exam.

Here the premises are

$(\exists x)\,[C(x) \wedge \sim R(x)]$, $(\forall x)\,[C(x) \to P(x)]$

Conclusion:-

$(\exists x)\,[P(x) \wedge \sim R(x)]$

1.	$(\exists x)[C(x) \wedge \sim R(x)]$	P
2.	$C(y) \wedge \sim R(y)$	Rule ES
3.	$C(y)$	T $P \wedge Q \Rightarrow P$
4.	$(\forall x)[C(x) \rightarrow P(x)]$	P
5.	$C(y) \rightarrow P(y)$	Rule US
6.	$P(y)$	T (3, 5)
7.	$\sim R(y)$	T (2) $P \wedge Q \Rightarrow Q$
8.	$P(y) \wedge \sim R(y)$	T (6, 7) $P, Q \Rightarrow P \wedge Q$
9.	$(\exists x)(P(x) \wedge \sim R(x))$	Rule EG.

Show that the premises, one student in this class know how to write program in JAVA. and Everyone who knows how to write programs in JAVA can get high paying job. imply the conclusion someone in this class can get a high paying job.

Sol:

$C(x)$: x is in this class.

$J(x)$: x is knows JAVA program.

$H(x)$: x is high paying job.

$$(\exists x)[C(x) \wedge J(x)],\ (\forall x)[J(x) \rightarrow H(x)]$$

conclusion,

$$(\exists x)[C(x) \wedge H(x)]$$

1.	$(\exists x)[C(x) \wedge \cancel{H}J(x)]$	P
2.	$C(y) \wedge \cancel{H}J(y)$	Rule ES
3.	$C(y)$	T $P \wedge Q \Rightarrow P$
4.	$(\forall x)[J(x) \rightarrow H(x)]$	P
5.	$J(y) \rightarrow H(y)$	Rule US
6.	$C(y) \wedge H(y)$	T $(^{2,4}_{3,5})$
7.	$(\exists x)C(x) \wedge H(x)$	Rule EG ~~T $P \wedge Q \Rightarrow Q$~~
8.	$H(y) \wedge J(y)$	T (6,7) $P, Q \Rightarrow P \wedge Q$
9.	$(\exists x)[H(x)$	

Using CP rule, show that $(x)[P(x) \to Q(x)] \Rightarrow (x)P(x) \to (x)Q(x)$

Sol:

Steps	premises	justification
1.	$(x)[P(x) \to Q(x)]$	P
2.	$P(y) \to Q(y)$	Rule US
3.	$(x)\ P(x)$	Assume
4.	$P(y)$	Rule US
5.	$Q(y)$	Rule T (3, 4)
6.	$(x)\ Q(x)$	Rule UG
7.	$(x)\ P(x) \to (x)Q(x)$	Rule CP.

not Show that $(\forall x)[P(x) \vee Q(x)] \Rightarrow (x)\ P(x) \vee (\exists x)\ Q(x)$

Sol:

We shall use the indirect method of proof by assume negation

$\sim[(x)[P(x) \vee Q(x)]]$ as an additional premises.

Steps	premises	justification.
1.	$\sim[(x)[(P(x) \vee Q(x))]]$ $\exists(x)$	Assume
2.	$\sim(x)P(x) \wedge \sim(\exists x)Q(x)$	T
3.	$\sim(x)P(x)$	T $(P \wedge Q = P)$
4.	$(\exists x) \sim P(x)$	T (3)
5.	$\sim(\exists x)Q(x)$	T $(P \wedge Q = Q)$
6.	$(\forall x) \sim Q(x)$	T (5)
7.	$\sim P(y)$	Rule ES (4)
8.	$\sim Q(y)$	Rule (US) 5
9.	$\sim P(y) \wedge \sim Q(y)$	T $(P, Q = P \wedge Q)$
10	$\sim(P(y) \vee Q(y))$	T
11.	$(x)[P(x) \vee Q(x)]$	P
12.	$P(y) \vee Q(y)$	Rule US
13.	$[P(y) \vee Q(y)] \wedge \sim[P(y) \vee Q(y)]$	T $P, Q = P \wedge Q$

Verify the validity of the following argument. Lions are dangerous animals. There are lions. Therefore there are dangerous animals.

Sol:

Let

$L(x) \to x$ is a lion

$D(x) \to x$ is a dangerous animal.

Premises are,

$$(\forall x)[L(x) \to D(x)], (\exists x)L(x)$$

Conclusion:

$$(\exists x)(D(x))$$

1.	$\forall x[L(x) \to D(x)]$	P
2.	$L(y) \to D(y)$	Rule US
3.	$(\exists x)L(x)$	P
4.	$L(y)$	Rule ES
5.	$D(y)$	T (2, 4)
6.	$(\exists x)D(x)$	EG

Show that $\sqrt{2}$ is irrational.

Sol:

Suppose $\sqrt{2}$ is rational

(i.e)., $\sqrt{2} = \frac{p}{q}$, $q \neq 0$, p and q have no common factor.

squaring on b/s,

$$\frac{p^2}{q^2} = 2$$

$$\Rightarrow p^2 = 2q^2$$

Since p^2 is even integer, p is an even.

$\therefore$ $p = 2m$ for some integer m

Since q^2 is even, q is even integer.

$\therefore$ $q = 2k$ for some integer k

$\therefore$ p and q are even, hence they have a common factor 2. which is a contradiction to our assumption. Therefore,

$\therefore$ $\sqrt{2}$ is irrational.