ARUNAI ENGINEERING COLLEGE

TIRUVANNAMALAI-03

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

YEAR/SEM: II/III

ODD SEM-2021

EC8395- COMMUNICATION ENGINEERING

NOTES

EC8395 COMMUNICATION ENGINEERING

UNIT I ANALOG MODULATION

Amplitude Modulation – AM, DSBSC, SSBSC, VSB – PSD, modulators and demodulators – Angle modulation – PM and FM – PSD, modulators and demodulators – Superheterodyne receivers

UNITII PULSE MODULATION

Low pass sampling theorem – Quantization – PAM – Line coding – PCM, DPCM, DM, and ADPCM And ADM, Channel Vocoder - Time Division Multiplexing, Frequency Division Multiplexing

UNIT III DIGITAL MODULATION AND TRANSMISSION

Phase shift keying – BPSK, DPSK, QPSK – Principles of M-ary signaling M-ary PSK & QAM – Comparison, ISI – Pulse shaping – Duo binary encoding – Cosine filters – Eye pattern, equalizers

UNIT IV INFORMATION THEORY AND CODING

Measure of information – Entropy – Source coding theorem – Shannon–Fano coding, Huffman Coding, LZ Coding – Channel capacity – Shannon-Hartley law – Shannon's limit – Error control codes – Cyclic codes, Syndrome calculation – Convolution Coding, Sequential and Viterbi decoding

UNIT V SPREAD SPECTRUM AND MULTIPLE ACCESS

PN sequences – properties – m-sequence – DSSS – Processing gain, Jamming – FHSS – Synchronisation and tracking – Multiple Access – FDMA, TDMA, CDMA.

TEXT BOOKS:

1. H Taub, D L Schilling, G Saha, ―Principles of Communication Systems‖ 3/e, TMH 2007
2. S. Haykin ―Digital Communications‖ John Wiley 2005

REFERENCES:

1. B.P.Lathi, ―Modern Digital and Analog Communication Systems‖, 3rd edition, Oxford University Press, 2007
2. H P Hsu, Schaum Outline Series – ―Analog and Digital Communications‖ TMH 2006
3. B.Sklar, Digital Communications Fundamentals and Applications‖ 2/e Pearson Education 2007.

EC8395-COMMUNICATION ENGINEERING

UNIT I

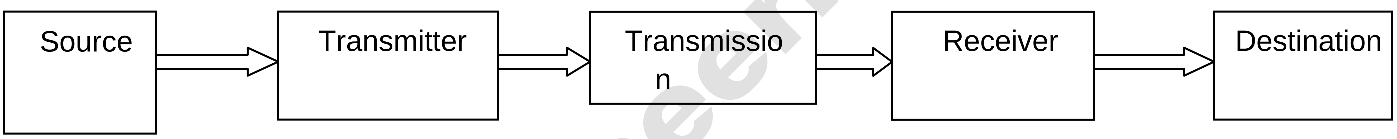
ANALOG COMMUNICATION

Communication is the process of conveying or transferring of information from one point to another point. Information can be image, text or any other data. Communication between any two points or places requires a medium in between them. This medium can be wired or wireless medium. The information that needs to be transmitted may not be in a form that is suitable for all medium. It needs to be processed. This processing of raw information to convert it into a form that is suitable for a medium is called as modulation.

TYPES OF COMMUNICATION

Communication can be of two types- Analog & Digital

In Analog communication the transmit information is continuous in nature, whereas in digital communication it is discrete in nature.



Basic block diagram for communication System

Information from the source should be modulated before transmission to enable proper transmission of information from transmitter to receiver. The process of modifying or changing any characteristics of any signal is called modulation. Any signal has three characteristic they are

1. Amplitude
2. Frequency
3. Phase

MODULATION:

The process of changing any one of the characteristics of carrier signal with respect to information signal is known as modulation.

TYPES OF MODULATION

Modulation is classified into two types they are, Analog modulation and Digital modulation. In analog modulation both information and carrier signal are analog in nature, whereas in digital information signal is digital but carrier signal is analog.

Modulation

Analog modulation

AM FM PM

Digital modulation

ASK FSK PSK QAM

AM (Amplitude modulation):

It is the process of changing the amplitude of high frequency carrier signal in accordance with low frequency information signal. Here frequency and phase angle of carrier remains unchanged.

FM (Frequency modulation):

It is the process of changing the frequency of carrier signal accordance with amplitude of information signal.

PM (Phase modulation):

It is the process of changing the phase of carrier signal accordance with amplitude of information signal..

NEED FOR MODULATION

- Reduce height of antenna
- Transmit signal over a long distance
- Avoid noise and interference
- Multiplexing
- Improve the signal to noise ratio

AMPLITUDE MODULATION

It is the process of changing the amplitude of a relatively high frequency carrier signal in proportion with the instantaneous value of the modulating signal. AM is used for commercial broadcasting of audio and video signals.

Applications of AM: 1. Two-way mobile radio, Audio and video broadcast

AM VOLTAGE DISTRIBUTION

The modulating signal is represented as, $e_m(t) = E_m \sin\omega_m t$

The carrier signal is represented as, $e_c(t) = E_c \sin\omega_c t$

According to the definition, the amplitude of the carrier signal is changed after modulation.

$$E_{AM} = E_c + e_m(t) = E_c + E_m \sin\omega_m t \text{------------------- (1)}$$

$$= Ec\,[1 + (E_m/E_c)\sin\omega_m t\,] \text{----------------------(2)}$$

$$E_{AM} = E_c(1 + m_a \sin\omega_m t\,) \text{----------------------------(3)}$$

Depth of Modulation/Modulation Index:

Coefficient of modulation and percent modulation:

If is defined as the ratio of maximum amplitude of the message signal to the maximum amplitude of the carrier signal.

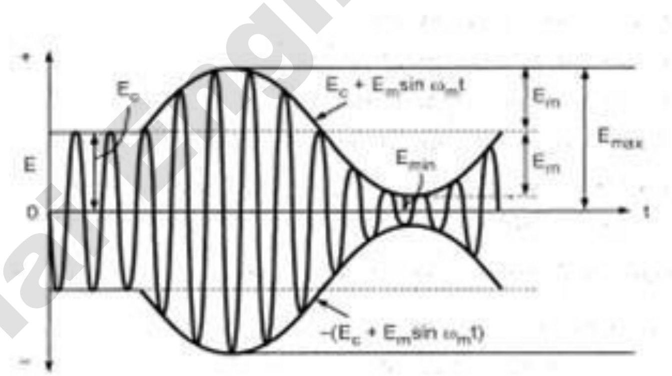
$$m_a = \frac{E_m}{E_c}$$

Percent modulation is indicated as M

$$M = \frac{E_m}{E_c} \times 100 \quad \text{or} \quad M = m_a \times 100$$

Relationship between m, E_m & E_c

From the figure.



$$E_m = \tfrac{1}{2}(E_{max} - E_{min})$$

$$E_C = \tfrac{1}{2}(E_{max} + E_{min})$$

$$M = \frac{\tfrac{1}{2}(E_{max} - E_{min})}{\tfrac{1}{2}(E_{max} + E_{min})} \times 100 = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100$$

Where $m_a = E_m/E_c$ Where m_a is the modulation index (or) depth of modulation. The value E_m must be less than value of E_c to avoid distortion in the modulated signal. Hence maximum value of m_a will be equal to 1. When m_a is expressed in percentage it is called percentage modulation.

But the instantaneous amplitude of modulated signal, i.e at any time

$$e_{AM}(t) = E_{AM} \sin\omega_c t \text{ ----------------------------------- } (4)$$

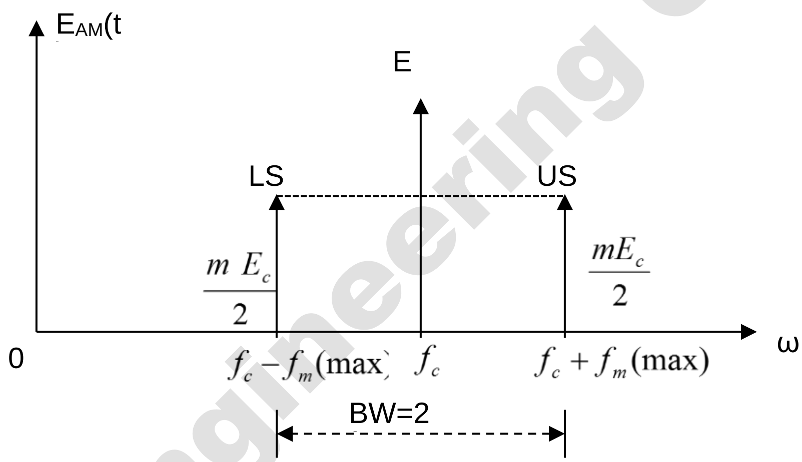
Substitute equation (3) in (4)

$$e_{AM}(t) = E_c (1 + m_a \sin\omega_m t)\sin\omega_c t$$

$$= E_c \sin\omega_c t + m_a E_c \sin\omega_m t \sin\omega_c t$$

(1) (2)

<u>Frequency spectrum and Bandwidth of AM Wave</u>



- The figure shows the frequency spectrum of Am.
- It extends from $f_c - f_m$ (max) to $f_c + f_m$ (max).
- The band of frequencies b/w f_c and $f_c - f_m$ (max) is called lower side band [LSB] and any frequency within this band is called lower side frequency [LSF].
- The band of frequencies b/w f_c and $f_c + f_m$ (max) is called upper side band [USB] and any frequency within this and is called upper side frequency [USF]

Bandwidth of AM.

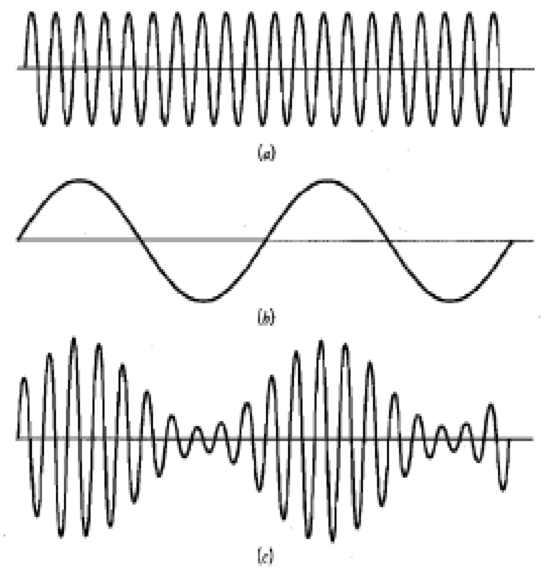
The Bandwidth of Am wave is equal to the difference b/w the highest upper side frequency and lowest lower side frequency.

$$B = f_c + f_m(\max) - [f_c - f_m(\max)]$$

$$= \quad f_c + f_m\,(max) - f_c + f_m\,(max)$$

$$BW \;=\; 2f_m\,(max)$$

AM Waveform



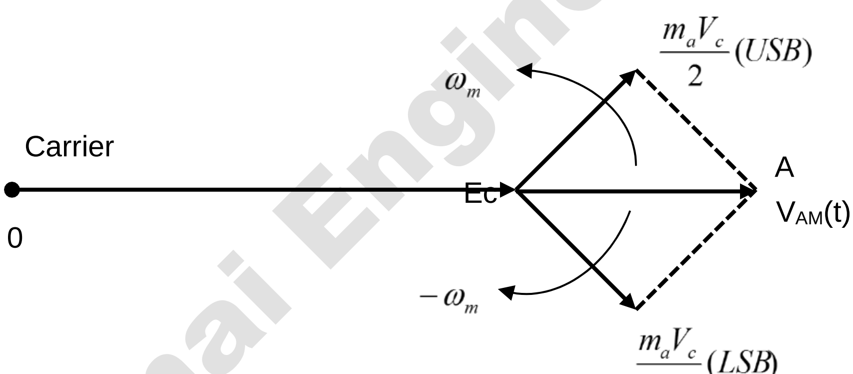
a) Message signal

b) Carrier signal

c) Amplitude modulated signal.

The shape of modulated waveform is known as **AM envelope.**

Phasor representation of AM



- **Based on the modulation index modulation can be either**,

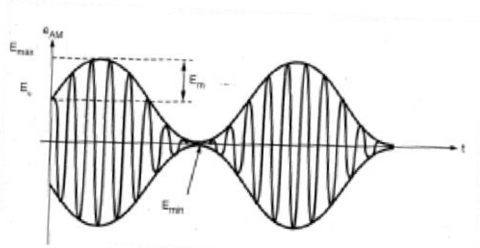
  (i). Critical Modulation

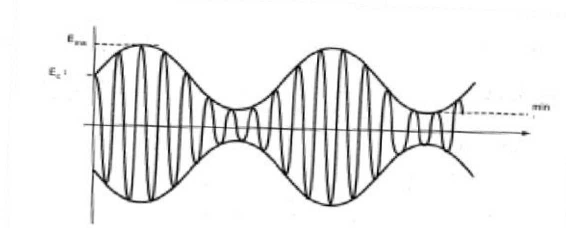
  (ii). Over Modulation

  (iii). Under Modulation

- When $E_m = E_c$ modulation goes to 100% this situation is known as critical modulation.

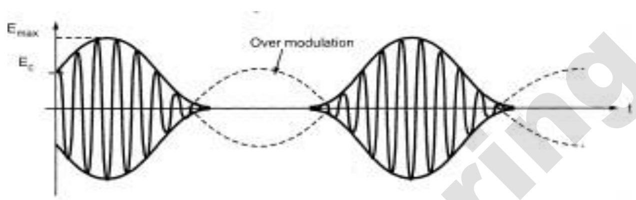
- $E_m < E_c$ leads to under modulation.





AM wave for m = 0.5

- $E_m > E_c$ leads to over modulation.



Envelope of AM wave for m = 3/2

AM POWER DISTRIBUTION

- The modulated wave contains three terms such as carrier wave, LSB, USB.
- The modulated wave contains more power than the unmodulated carrier.

Total Power in modulated wave will be,

$$P_t = P_C + P_{USB} + P_{LSB}$$

i.e. total power P_t of AM wave is the sum of carrier power and side band power.

P_C - Carrier power, P_{USB} - Upper Side Band power, P_{LSB} - Lower Side Band power

The carrier power is

$$P_c = \frac{E_c^2}{R} = \frac{(Ec / \sqrt{2})^2}{R} = \frac{Ec^2}{2R}$$

The lower and upper sideband powers are same which is given by

$$P_{USB} = P_{LSB} = \frac{(m_a E_c/ 2\sqrt{2})^2}{R} = \frac{m^2_a E_c^2}{8R}$$

In terms of carrier power

$$P_{USB} = P_{LSB} = \frac{(m_a E_c/ 2\sqrt{2})^2}{R} = \frac{m^2_a E_c^2}{8R} = \frac{m^2_a P_c}{4}$$

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$P_t = P_c + \frac{m_a{}^2 P_c}{4} + \frac{m_a{}^2 P_c}{4}$$

$$P_t = P_c \left(1 + \frac{m_a{}^2}{2}\right)$$

AM CURENT DISTRIBUTION

The equation of P_t given by

$$\frac{P_t}{P_c} = \left(1 + \frac{m_a{}^2}{2}\right)$$

$$P_t = I_t^2 R$$

$$P_c = I_c^2 R$$

$$I_t^2 = I_c^2 \left(1 + \frac{m_a{}^2}{2}\right)$$

$$I_t = I_c \sqrt{1 + \frac{m_a{}^2}{2}}$$

Where I_t = Total(or)modulated current, I_C = Carrier current

Efficiency:

$$\% \eta = \frac{\text{Power in side band}}{\text{Total Power}} * 100$$

$$= \frac{P_{LSB} + P_{LSB}}{P_{Total}} * 100$$

$$= \frac{m_a{}^2}{2 + m_a{}^2} * 100$$

If $m_a = 1$, Then $\%\eta = 33.33\ \%$

In this only one third of total power is carried by the sidebands and the rest two third is wasted.

Modulation by a complex information signal:

- If the modulating signal contains two frequencies say f_{m1} & f_{m2}, the modulated wave will contain the carrier and two sets of side frequencies. Spaced symmetrically about the carrier.

$$V_{am}(t) = \sin 2\pi f_c t + \frac{1}{2} \sin 2\pi (f_c - f_{m1}) t - \frac{1}{2}\sin 2\pi (f_c + f_{m1}) t + \frac{1}{2}\sin 2\pi (f_c - f_{m2}) t - \frac{1}{2}\sin 2\pi (f_c + f_{m2}) t$$

- The coefficient of modulation or modulation index is given by
  $m_t = \sqrt{m_1^2 + m_2^2}$
- In general for n different signals $m_t = \sqrt{m_1^2 + m_2^2 + \ldots + m_n^2}$
- $P_{usbt} = P_{lsst} = P_c m^2 / 4$

$$P_t = P_c (1 + m^2 / 2)$$

Advantages, Disadvantages and Applications of AM (DSBFC)

Advantages

1. Simple and inexpensive receivers. Easy to detect with simple equipment even if the signal is not very strong
2. Narrow bandwidth than FM
3. Wider coverage
4. Well-established, mature art used for broadcasting almost exclusively

Disadvantages

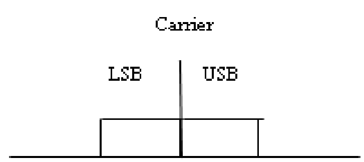
1. Received signal affected by electrical storms and other radio frequency interference
2. Receivers able to reproduce frequencies up to 5 MHz or less
3. Inefficient use of transmitter power

Applications

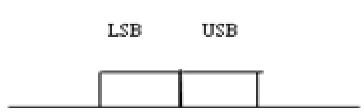
1. Low quality form of modulation that is used for commercial broadcast of both audio and video signals
2. Two way mobile radio communications such as citizen band (CB) radio
3. Aircraft communication in the VHF frequency range

TYPES OF AM MODULATION

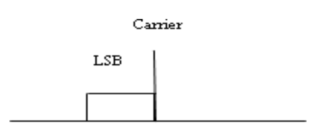
1. Double sideband with full carrier



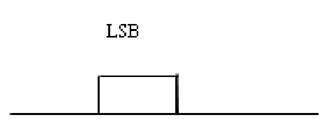
2. Double side without carrier



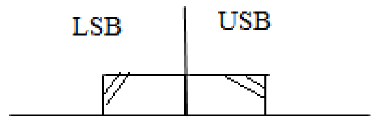
3. Single sideband with carrier



4. Single sideband without carrier



5. Vestigial sideband



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AM modulating Circuits

- Based on the location in the transmitter AM modulating circuits are classified as
  a. Low level AM modulator
  b. High level AM modulator

Difference b/w low level and High level AM mod.

Low level AM modulator	High level AM modulator
Modulation takes place prior to the final stage of the transmitter	Modulation takes place in the final element of final stage.
Less modulating signal power is required	More modulating signal power is required.

GENERATION OF AM-DSBFC (AM MODULATORS)

The generation method of AM waves are broadly divided in to two types

- Linear modulator(or)large signal modulator(or)high level modulation
- Non Linear modulator(or)small signal modulator(or)low level modulation

LINEAR MODULATOR:

In this type of modulators the devices are operated in linear region of its transfer characteristics. **Linear modulators are also divided in to two types,**

- Transistor modulator
- Switching modulator.

NON LINEAR MODULATORS:

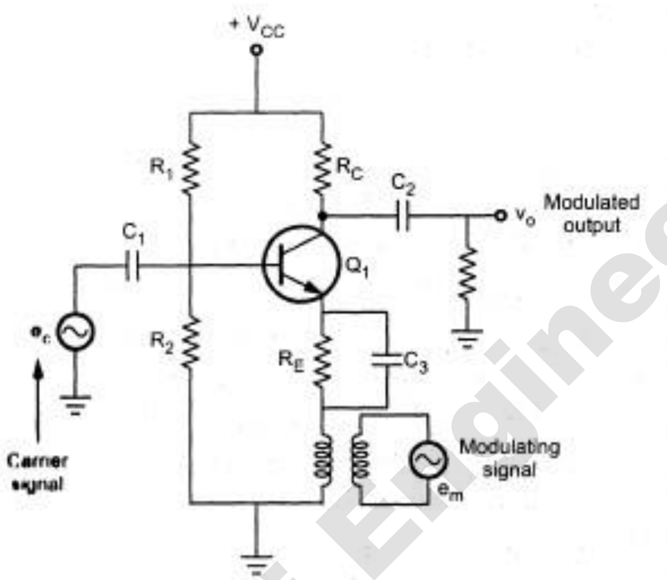
These modulators are operated in nonlinear region.These are used in low level modulation.

The types of non linear modulators are,

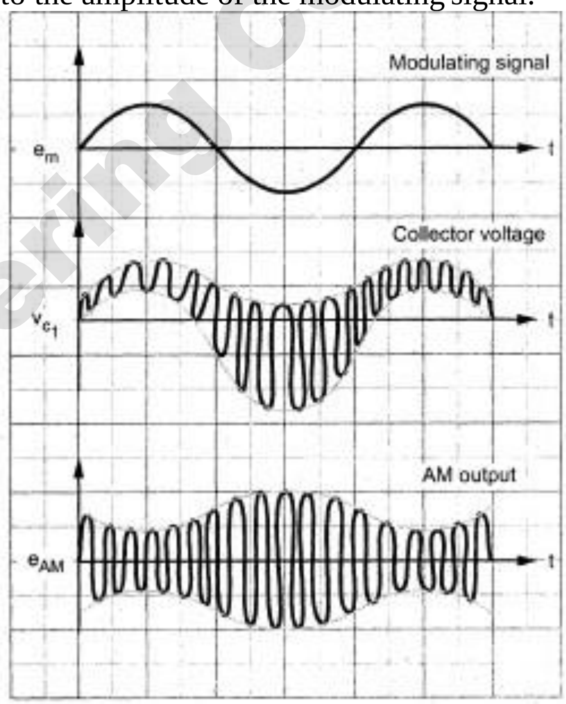
Square law modulator,
Product modulator,
Balanced modulator.

LOW LEVEL AM MODULATOR

- Class A amplifier can perform amplitude modulation.
- Amplifier must have 2 inputs one for the carrier signal & second for modulating signal.
- With no modulating signal present, the circuit operates as a linear class A amplifier, and the output is simply the carrier amplified by the quiescent voltage gain .
- The carrier is applied the base and the modulating signal to the emitter. Hence it is also called as **Emitter Modulation**.
- The modulating signal varies the gain of the amplifier at a sinusoidal rate equal to the frequency of the modulating signal.
- The depth of modulation achieved is proportional to the amplitude of the modulating signal.



(a) Circuit diagram of low level AM modulator



(b) Waveforms

- The voltage gain for an emitter modulator is expressed as,

$A_v = A_q (1 + m \sin 2\pi fmt)$

A_v $\rightarrow$ Amplifier voltage gain with modulation.

A_q $\rightarrow$ Amplifier quiescent (without modulation) voltage gain.

$\sin 2\pi f_m t$ varies from + 1 to -1

Hence, $A_v = A_q (1 \pm m)$

Av_{max} = 2Aq, when m = +0

Av_{min} = 0, when m = -1

- The modulating signal drives the circuit into both saturation and cutoff, thus producing the nonlinear amplification necessary for modulation to occur. The collector waveform includes the carrier and the upper and lower side frequencies as well as a component at the modulating signal frequency from the waveform, thus producing a symmetrical AM envelop at V_{out}.

Advantages of Low level modulation:

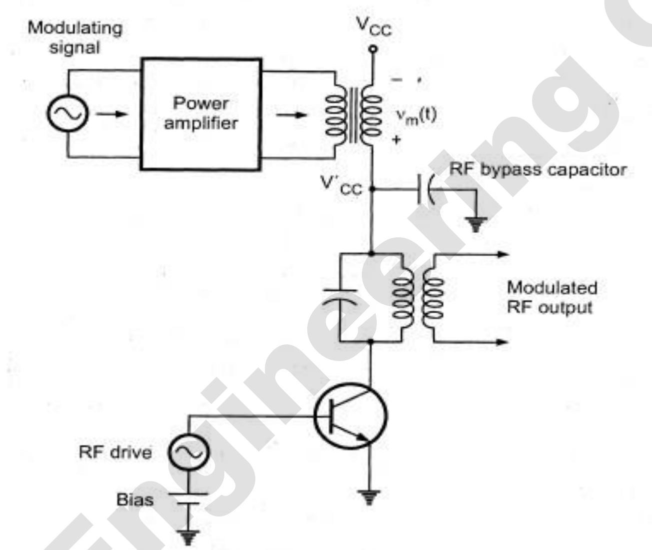
1. Less modulating signal power is required to obtain high percentage modulation.
2. Modulating circuit is designed for low power.

Disadvantage of Low level modulation

Amplifiers following modulator stage must be linear. At high operating powers linear amplifiers are very inefficient.

HIGH POWER AM MODULATOR / MEDIUM POWER AM MODULATOR

- The class C amplifier is used. It operates nonlinear and is capable of nonlinear mixing (modulation).
- This is known as collector modulator because modulating signal is applied to the collector.
- When the amplitude of the carrier exceeds the barrier potential (0.7V) Q1 turns on collector current flows.



- When carrier voltage drops below 0.7V Q1 turns off and collector current cases.
- The corresponding current and voltage waveforms are shown.
- When the modulating signal is applied it adds up with the Ec_c and gets submitted from Ec_c producing an Am o/p.

Advantages of high level modulators:

- There is no constraint of linear operation on amplifiers preceding modulator stage.
- Power efficiency is good

Disadvantages of high level modulators:

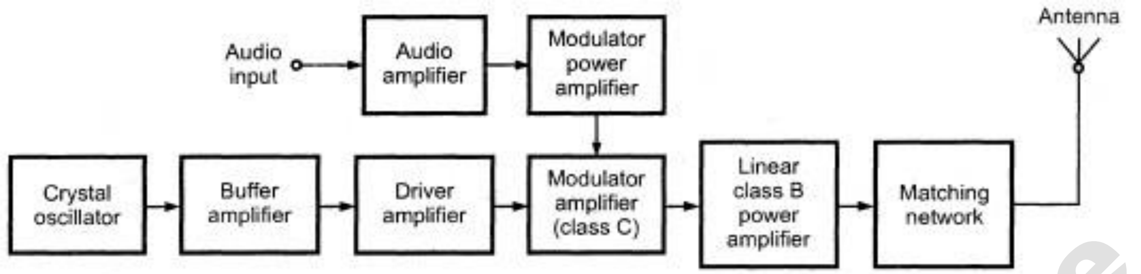
- High modulating power is required.
- Final modulating signal amplifier has to supply all the sideband power.

AM TRANSMITTERS

Low Level AM Transmitter

- The block diagram of a typical AM transmitter is shown in which carrier source is a crystal oscillator. The crystal oscillator is stabilized in order to maintain the carrier frequency deviation within a prescribed limit.

- The crystal oscillator is followed by a tuned buffer amplifier (class B) and drive amplifier.

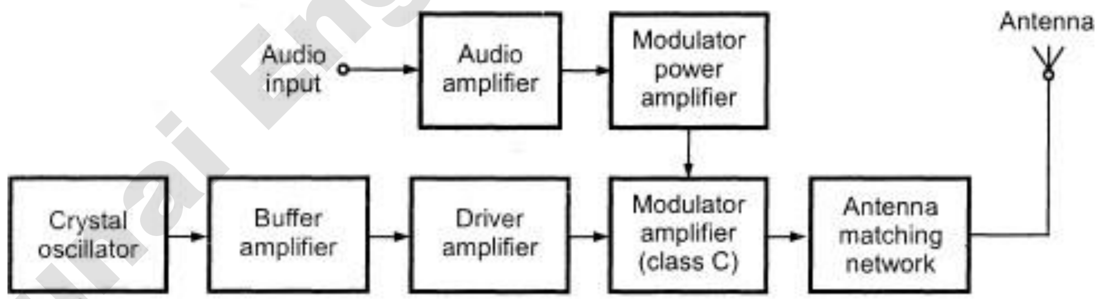


Low level AM Transmitter

- The modulator circuit used is generally a class C power amplifier that is a collector modulator.
- The audio signal is amplified by a chain of low level audio amplifiers and a power amplifier. This amplifier is controlling the power being delivered to the final RF amplifier. Class B push pull amplifier is usually used for this purpose.
- The amplified modulating signal is applied to the modulator along with the carrier. AM wave is got at the output of the modulator.
- This AM signal is then amplified using a chain of linear amplifiers to raise its power level. Class B amplifiers are used for this purpose. The linear power amplifier is used to avoid the distortion in the AM wave.
- The amplitude modulated signal is then transmitted using transmitting antenna. The matching network matches the output impedance of the final amplifier to the transmission line and antenna.

Application of Low Level AM Transmitter: It is used in low capacity system such as wireless intercoms, Remote control units, pagers, shot range talkie.

High Level AM Transmitter



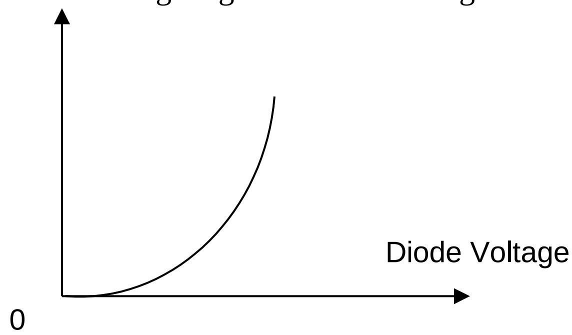
High level AM Transmitter

- Here modulating signal power should be higher than the low level.
- The amplification takes place prior to modulation.

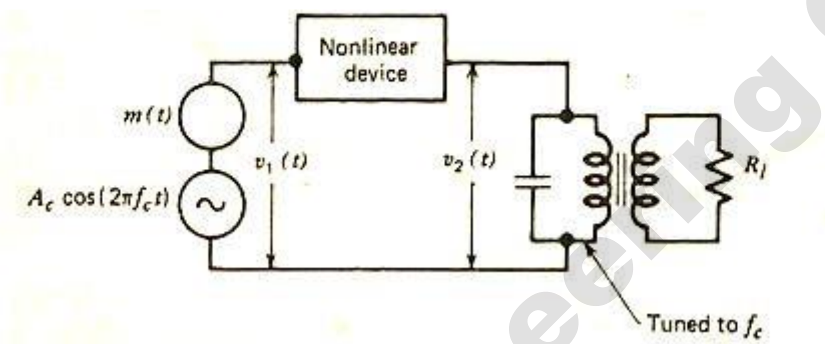
Application of High Level Transmitter: Used for long distance communications.

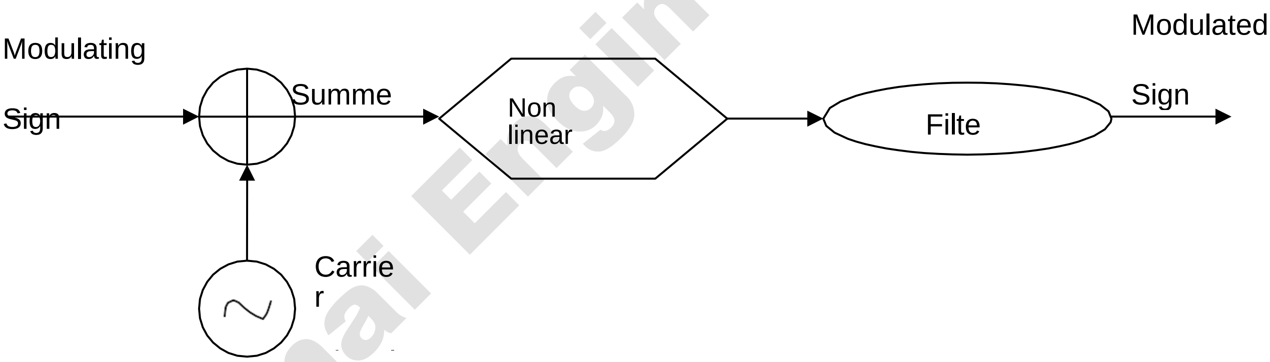
SQUARE LAW MODULATOR

- The non linear portion of V-I characteristics of diode is used as a element for non-linear modulators.
- This is suited at low voltage levels because of the fact that current-voltage characteristics of a diode is highly popuularly in the low voltage region as shown in figure.



- A square law modulator has three features shown in figure.
- Summer - To sum carrier and modulating signal
- A non linear element
- Band pass filter for extracting desired modulating products.





Principle of operation:

- From the figure the non linear device diode is used to produce low level amplitude modulation.
- Here the carrier and modulating signals are applied across the diode.
- A dc battery Ecc is connected across the diode to get a fixed operating point on the V-I characteristics of the diode.
- This amplitude modulation that is low level can be explained by considering the fact when two different frequencies are passed through the non linear device.
- So when we apply carrier and modulating frequencies different frequency terms appear at the output of diode.
- These different frequency terms are applied across the tuned circuit which is tuned to carrier frequency and has a narrow bandwidth just to pass two sidebands along with the carrier and reject other frequencies.

- So the output of the tuned circuit will be carrier and two sidebands that is amplitude modulated wave is produced.

<u>Mathematical analysis:</u>

Let the modulating voltage will be

$$e_m(t)= E_m \sin\omega_m t \text{ -- } (1)$$

Let the carrier voltage will be

$$e_c(t)= E_c \sin\omega_c t \text{ -- } (2)$$

So the input voltage applied to the diode will be

$$e_1(t) = e_m+ec$$
$$= E_m \sin\omega_m t+ E_c \sin\omega_c t \text{ ------------------------ } (3)$$

We know that the current and voltage relationship of a linear circuit is,

$$i=av$$

where i=current through linear device
v=voltage through the linear device
a=proportionality constant

- The current and voltage relationship of a nonlinear circuit may be expressed as,

$$i=a V_1+b V_1^2+c V_1^3+ \text{ - - - - - - - - - -}$$

- This means that due to non-linearity in the V-I characteristics of a non-linear circuits, the current becomes proportional not only to voltage but also to the square, cube and higher powers of voltage. So

$$i= a_1V_1+a_2 V_1^2+a_3 V_1^3+ \text{------------------------------} (4)$$

Where a_1, a_2, a_3 are proportionality constants and V_1 is the input voltage applied to the diode or FET.

- Sub eqn (3) in (4)then

$$i= a_1(E_m \sin\omega_m t+ E_c \sin\omega_c t)+ a_2(E_m \sin\omega_m t+ E_c \sin\omega_c t)^2+\text{------------}$$
$$= a_1 Em \sin\omega_m t+ a_1 Ec \sin\omega_c t+ a_2 Em^2 \sin\omega_m^2 t+ a_2 Ec^2 \sin\omega_c^2 t+ 2a_2EmEc \sin\omega_m t\sin\omega_c t+\text{---}$$

Neglecting second and higher order terms, we get

$$i(t)= a_1 Em \sin\omega_m t+ a_1 Ec \sin\omega_c t +a_2EmEc \sin\omega_m t \sin\omega_c t+\text{---}$$

$$= a_1 Em \sin\omega_m t+ a_1 Ec \sin\omega_c t +a_2EmEc [\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t]$$

- The tuned circuit is tuned to the carrier frequency and it allows only ω_c, $\omega_c+\omega_m$, $\omega_c-\omega_m$ terms and eliminates all other terms.hence we obtain

$$i(t)= a_1 Ec \sin\omega_c t +a_2EmEc [\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t]$$

$$\boxed{i(t)= \underbrace{a_1 Ec \sin\omega_c t}_{\text{(carrier)}} +\underbrace{a_2EmEc \cos(\omega_c-\omega_m)t}_{\text{(LSB)}} - \underbrace{a_2EmEc \cos(\omega_c+\omega_m)t}_{\text{(USB)}} \text{-------} (5)}$$

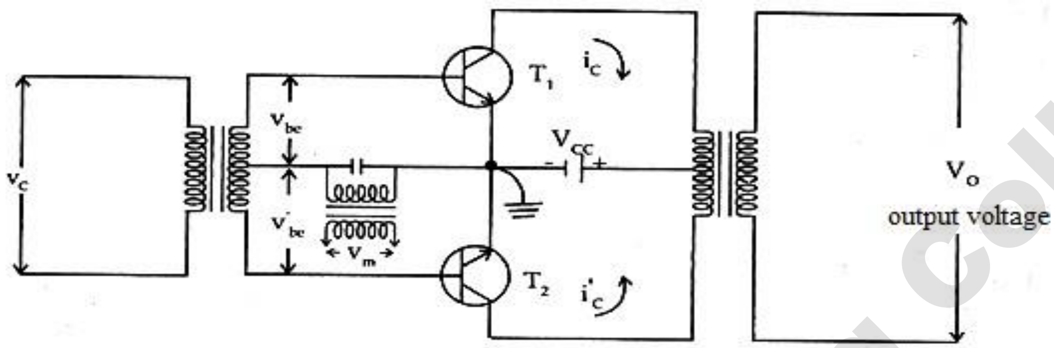
- The main drawback of using diode modulator is it does not provide amplification and a single diode is unable to balance out the frequency completely. These limitations can be eliminated by using amplifying devices like transistor, FET in a balanced mode.

- We can also use the square law modulator with FET instead of diode.

- From the final equation we know that it consists only the carrier and USB and LSB frequency components and all the components are removed.

BALANCE MODULATOR METHOD

It is assumed that the two transistors are identical and the circuit is symmetrical. The operation is confined in the nonlinear region of the active devices employed in this circuit. The carrier voltage across the upper and lower part of the secondary windings of the center tap transformers are equal in magnitude and opposite in phase.



BALANCED MODULATOR

- In this the carrier voltage across the two windings of a centre tap transformers are equal and opposite in phase $V_{be}= -V'_{be}$
- Input voltage to the transistor T1 is given by,

  $V_{be} = e_c(t) + e_m(t)$

  $V_{be} = E_c \sin\omega_c t + E_m \sin\omega_m t$ --(1)

- Similarly Input voltage to the transistor T2,

  $V'_{be} = -e_c(t) + e_m(t)$

  $V'_{be} = -E_c \sin\omega_c t + E_m \sin\omega_m t$---(2)

By using the non-linearity property the collector current can be written as per square law equation

$i_c = a_1 V_{be} + a_2 V_{be}^2 + \ldots\ldots$ ---(3)

$i_c' = a_1 V'_{be} + a_2 V'^2_{be} + \ldots\ldots$ ---(4)

On substituting eqn 1 and 2 in 3 and 4 we get,

$i_c = a_1 [E_c \sin\omega_c t + E_m \sin\omega_m t] + a_2 [E_c \sin\omega_c t + E_m \sin\omega_m t]^2$

$i_c = a_1 [E_c \sin\omega_c t + E_m \sin\omega_m t] + a_2 E^2_c \sin^2\omega_c t + a_2 E^2_m \sin^2\omega_m t + 2a_2 E_m E_c \sin\omega_m t \sin\omega_c t$ ----(5)

$i'_c = a_1 [-E_c \sin\omega_c t + E_m \sin\omega_m t] + a_2 [-E_c \sin\omega_c t + E_m \sin\omega_m t]^2$

$i'_c = a_1 [-E_c \sin\omega_c t + E_m \sin\omega_m t] + a_2 E_c^2 \sin^2\omega_c t + a_2 E_m^2 \sin^2\omega_m t - 2a_2 E_m E_c \sin\omega_m t \sin\omega_c t$ -- (6)

The output AM Voltage V_0 is given by $V_0 = K [i_c - i_c']$ --------------------------------------- (7)

since i_c, i'_c flows in the opposite direction and 'K' is a constant depending on impedance or other circuit parameters.

On substituting equation 5 and 6 in 7 we get,

$$V_o = K\,[\,i_c - i_c'\,]$$

$$E_o = K\,[2\,a_1\,E_c \sin\omega_c t + 4a_2\,E_m\,E_c \sin\omega_m t \sin\omega_c t]$$

$$= 2KE_c\,a_1 \sin\omega_c t \left(1 + \frac{2\,a_2}{a_1} E_m \sin\omega_m t\right)$$

$$= 2KE_c\,a_1 \sin\omega_c t \left(1 + m_a \sin\omega_m t\right)$$

where $m_a = \dfrac{2\,a_2\,E_m}{a_1}$ is the modulation index

This equation represents the output modulated voltage which contains carrier and side band terms but no modulating terms. Thus this circuit is used as AM modulator. The advantage of this circuit is that the undesired harmonics are automatically balanced out and hence there is no need for separate filter.

DEMODULATION/DETECTION OF AM-DSBFC (AM DEMODULATORS)

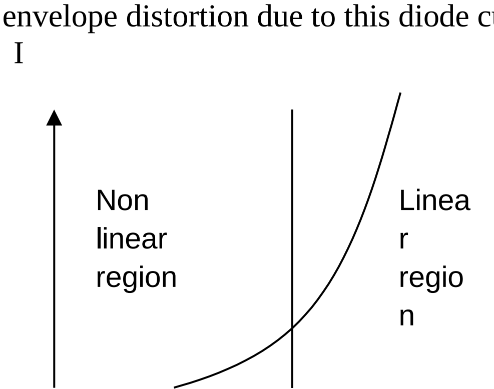
- Demodulation or detection is nothing but the process of extracting a modulating or information signal from modulated signal. Otherwise in other words, demodulation or detection is the process by which the message is recovered from the modulated signal at receiver.
- The devices used for demodulation or detection are called as demodulators or detector
- For amplitude modulation, the detectors or demodulators are categorized are

  Square law detectors or nonlinear detectors

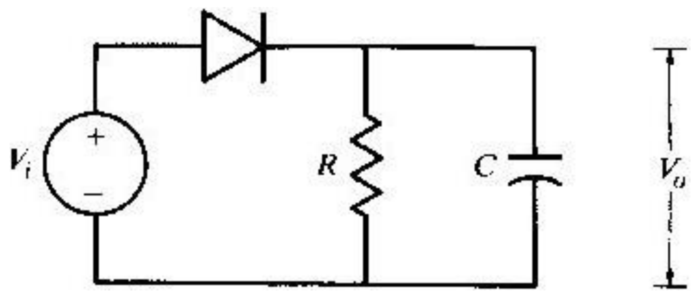
  Linear detectors
- The low level modulated signals are using non linear detectors to recover the original message signal

SQUARE LAW DETECTOR

- The square law detector circuit is used for detecting low level modulated signal.
- Here the diode is used in V – I characteristics of the device i.e. non linear characteristics of the diode.
- The square law detector is similar to the square law modulator. The only difference lies in the filter circuit.
- In square law modulator we are using band pass filters, in square law detector; a low pass filter is used. The operation is limited to the non-linear region of the diode characteristics, so the lower half portion of the modulated waveform is compressed.
- This produces the envelope distortion, so the average value of the diode – current is no longer constant and varies with time. The average diode current consists of steady DC component and time varying modulation frequency. Due to nonlinear region the lower half of its current wave form is compressed. This may cause envelope distortion due to this diode current will not be constant, and varies with time.



V



SQUARE LAW DETECTOR

The distorted output diode current is expressed by the non-linear V-I relationship (i.e. square law) is,

$$i_0 = a_1\, e_{AM} + a_2\, e_{AM}^{\ 2} \text{ -------------------------- } (1)$$

where e_{AM} =i/p modulated signal.

- The equation of AM wave is

$$e_{AM} = Ec\,(1 + m_a \sin\omega_m t)\sin\omega_c t \text{ --------------------------------- } (2)$$

On substituting equation 2 in 1 we get

$$i_0 = a_1\, Ec\,(1 + m_a \sin\omega_m t)\sin\omega_c t] + a_2[Ec\,(1 + m_a \sin\omega_m t)\sin\omega_c t\,]^2$$

$$i_0 = a_1\, Ec \sin\omega_c t + a_1 m_a\, Ec \sin\omega_c t \sin\omega_m t + a_2\, Ec^2 \sin^2\omega_c t\,(1 + m_a \sin\omega_m t)^2$$

$$= a_1\, Ec \sin\omega_c t + a_1 m_a\, Ec \sin\omega_c t \sin\omega_m t + a_2\, Ec^2 \sin^2\omega_c t(1 + m_a^2 \sin^2 \omega_m t + 2\, m_a \sin\omega_m t\,)$$

$$= a_1\, Ec \sin\omega_c t + a_1 m_a\, Ec \sin\omega_c t \sin\omega_m t + a_2\, Ec^2\,(1 + m_a^2 \sin^2 \omega_m t + 2\, m_a \sin\omega_m t\,)\left(\frac{1-\cos 2\omega_c t}{2}\right)$$

$$= a_1\, Ec \sin\omega_c t + a_1 m_a\, Ec \sin\omega_c t \sin\omega_m t + \frac{a_2\, Ec^2}{2} - a_2\, Ec^2\, \frac{\cos 2\omega_c t}{2} + \frac{a_2\, Ec^2\, m_a^2 \sin^2 \omega_m t}{2} +$$

$$- \frac{a_2\, Ec^2\, m_a^2 \sin^2\omega_m t \cos 2\omega_c t}{2} + a_2\, Ec^2\, m_a \sin\omega_m t - a_2\, Ec^2\, m_a \sin\omega_m t\, \cos 2\omega_c t$$

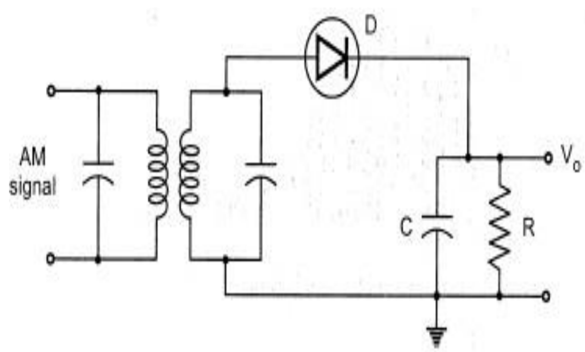
The carrier term is filtered by using a low pass filter and high frequency term are also reduced then,

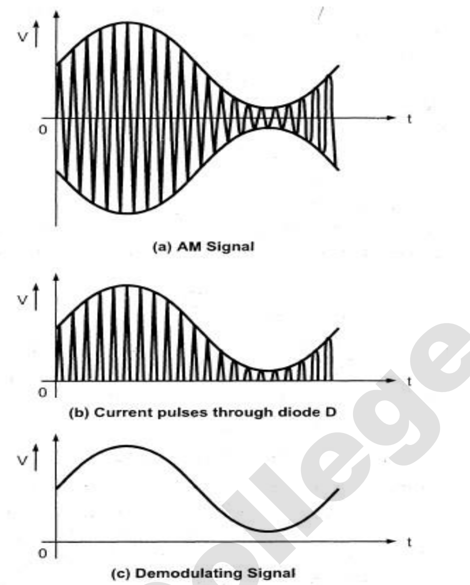
$$i_0 = a_2\, Ec^2\, m_a \sin\omega_m t$$

So the final output contains a signal with modulating frequency. Hence original signal is recovered.

ENVELOPE DETECTOR

- A detector circuit whose output follows the envelope of the modulated signal which is used to reproduce the modulating or message signal is called as “Envelope Detector”.
- This is most popular commercial receiver circuits since it is very simple and not expensive, and also it gives satisfactory performance.An envelope detector of the series type is shown in Figure which consists of a diode and a resistor capacitor filter a time constant network.





Principle Of Operation:

- Modulated signal is applied to the series combination of diode and the load impedance consisting of a resistor R and C.
- Operation takes place over the linear region of VI characteristics of Diode.
- For positive cycle of carrier signal the diode D conducts thereby the capacitor charges to the peak voltage with the time constant **τ =R C o**f the carrier signal through the resistor R.
- As the input falls below the peak value, the diode reaches cut-ff. The diode acts as open switch and hence the capacitor gets discharge path through R.
- During negative half cycle the diode is reverse biased and the carrier voltage is disconnected from the RC circuit. So the capacitor discharges continuously until next positive cycle appears.
- From the peak of one positive cycle to the next the capacitor discharges slowly and this process continues.Thus the voltage across ‘C’ is same as the envelope of the modulated carrier but spikes are introduced. So the output voltage across capacitor is a spiky modulating or base band signal. So the envelope is detected at the output of capacitor. Thus from the average value the original signal is recovered by extracting the envelope.
- The spikes can be reduced to a negligible amount by keeping the time constant RC large so that the capacitor C discharges negligible amount.

CHOICE OF TIME CONSTANT RC

- Large (or) small value of time constant makes problem. So time constant is important consideration.
- **If time constant RC is quite low: Discharge curve during non conductive period is almost vertical, so fluctuations may occur in output voltage. This results in Diagonal clipping**
- **If RC is very large: Discharge curve is almost horizontal, so several peaks will be missed in the rectified output voltage. This results in negative peak clipping.**

Distortion in diode detector:

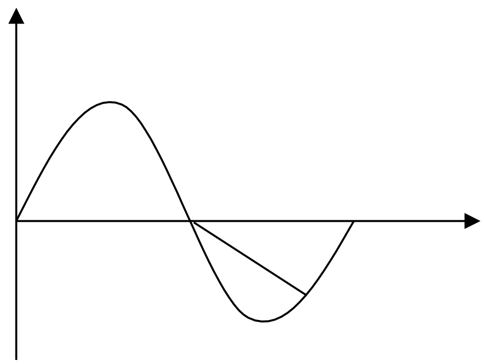
There are two clippings i.e. distortions available

Negative peak clipping

Diagonal clipping.

Diagonal clipping:

It results when time constant of detector is not selected properly. If the modulating voltage is faster than the rate of voltage fall across RC combination resulting in distorted output. This type of distortion is called diagonal clipping which results in distorted output.



- To avoid the diagonal clipping proper value of RC needs to be selected. The voltage across RC combination during the non-conducting period of diode is Vc at an instant t which is given by

$$V_C(t) = V_0 e^{-\frac{t}{RC}}$$

The rate of slope in capacitor due to period discharging is calculated by differentiating $V_C(t)$ So

$$\frac{dV_C(t)}{dt} = \frac{dV_C}{dt} = \frac{-V_0 e^{-\frac{t}{RC}}}{RC} = \frac{-V_C(t)}{RC} \quad \text{..................(1)}$$

The decrease in capacitor voltage must follow the modulation envelope if distortion is to be avoided. The envelope of the modulated voltage is given by

$$e_{AM} = E_C(1 + m_a \cos\omega_m t)$$

The slope of the envelope is given by

$$\frac{de_{AM}}{dt} = \frac{d}{dt}\left[E_C(1 + m_a\cos\omega_m t)\right]$$

$$= -m_a E_C \omega_m \sin\omega_m t \quad \text{--------------------------(2)}$$

To avoid the clipping the slope of capacitor voltage V_C should be algebraically equal or less than the slope of envelope voltage, so

$$-\frac{V_C(t)}{RC} \leq -E_C m_a \omega_m \sin\omega_m t$$

$$\frac{V_C(t)}{RC} \geq E_C m_a \omega_m \sin\omega_m t$$

$$\frac{E_C(1 + m_a\cos\omega_m t)}{RC} \geq E_C m_a \omega_m \sin\omega_m t$$

Mathematically the above equation is written as

$$\frac{1}{RC} \geq \frac{m_a \omega_m \sin\omega_m t}{(1 + m_a\cos\omega_m t)} \quad \text{------------------------(3)}$$

To find the maximum amount of RC differentiate the equation 3 and equate it to zero

$$\frac{d}{dt}\frac{m_a \omega_m \sin\omega_m t}{(1 + m_a\cos\omega_m t)} = 0$$

$$m_a \omega_m \frac{(1 + m_a\cos\omega_m t)\cos\omega_m t\,\omega_m - \sin\omega_m t\,(m_a(-\sin\omega_m t)\omega_m)}{(1 + m_a\cos\omega_m t)} = 0$$

$$\omega_m \cos\omega_m t + (m_a\cos^2\omega_m t)\omega_m + (m_a\sin^2\omega_m t)\omega_m = 0$$

$\omega_m \cos \omega_m t + m_a \omega_m (\cos^2\omega_m t + \sin^2\omega_m t) = 0$

$\omega_m \cos \omega_m t + m_a \omega_m = 0$

$\cos \omega_m t = - m_a$ --(4)

on substituting the value of $\cos\omega_m t$ in $\cos^2\omega_m t + \sin^2\omega_m t = 1$ we get $\sin\omega_m t = \sqrt{1-m_a^2}$ -------------(5)

substituting equation 4 and 5 in 3 we get $\frac{1}{RC} \geq \frac{m_a \omega_m \sqrt{1-ma2}}{1-m_a^2}$

By simplifying $RC \geq \frac{\sqrt{1-m_a^2}}{m_a \omega_m}$

So RC depends on the m_a value. So the RC to be selected by satisfying the condition in order to avoid distortion and the modulation index value should be correct, if it is large then it will provide negative peak clipping. So the envelope detector will be changed for smooth functioning by attaching a π section low pass filter at the output.

Negative peak clipping:

- The 2nd source of distortion in linear diode detector is the curvatures of the diode characteristics. So as a result the efficiency varies. It will be reduced by selecting load resistance value large. So when R_C is large then m_a will be low and signal becomes clipped at the negative peaks.
- The negative peak clipping provides ac and dc load impedances unequal.

DOUBLE SIDEBAND SUPPRESSED CARRIER (DSBSC) MODULATION

In AM with carrier scheme, there is wastage in both transmitted power and bandwidth. In order to save the power in amplitude modulation the carrier is suppressed because it does not contain any useful information. This scheme is called as the double side band suppressed carrier amplitude Modulation (DSB-SC). It contains LSB and USB terms, resulting in a transmission bandwidth that is twice the bandwidth of the message signal.

Let us consider the message and carrier signal as,

$e_m(t) = E_m \sin \omega_m t$ ------------ (1)

$e_c(t) = Ec \sin \omega_c t$ ------------- (2)

For obtaining the DSB-SC wave multiply both carrier signal and message signal hence,

$e(t)_{DSB-SC} = e_m(t).e_c(t)$

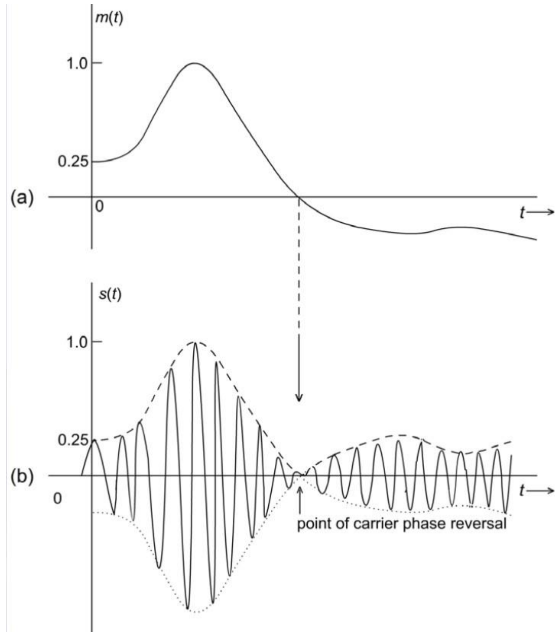
$= Em.Ec \sin\omega_m t \sin\omega_c t$

$e(t)_{DSB-SC} = \frac{Em.Ec}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$

USB LSB

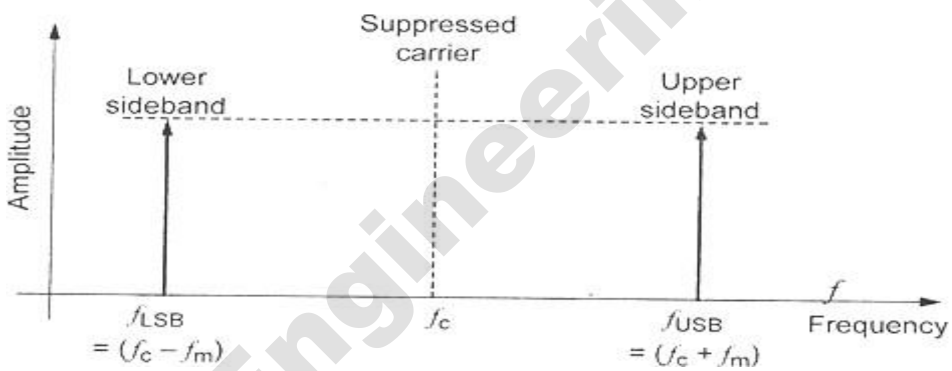
From this equation we know that the carrier is suppressed in double side band suppressed carrier.

Graphical Representation of DSB-SC AM:



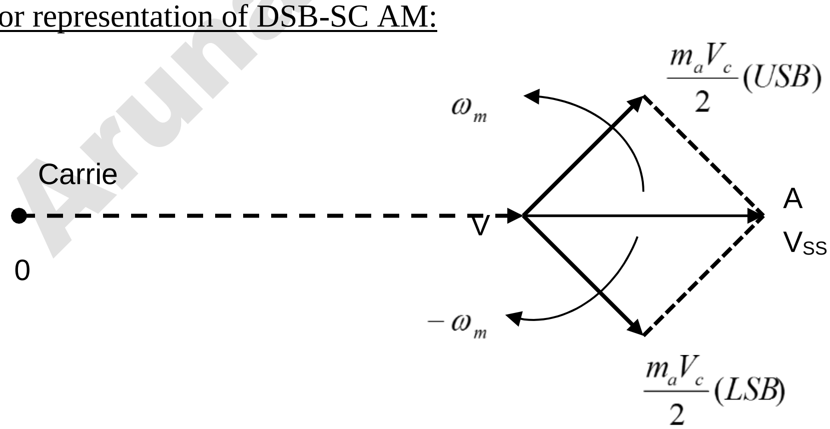
(a) Baseband signal (b) DSBSC

<u>Frequency spectrum of DSB-SC AM</u>



It shows that carrier term ω_c is suppressed. It contains only two sideband terms having frequency $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$. Hence this scheme is called as DSB-SC AM.

<u>Phasor representation of DSB-SC AM:</u>



<u>POWER CALCULATION:</u>

The total power transmitted in AM is

$$P_t = P_{carrier} + P_{LSB} + P_{USB}$$

If the carrier is suppressed , then the total power transmitted is

$$P_t' = P_{LSB} + P_{USB}$$

We know that, $P_{LSB} = P_{USB} = \frac{m_a^2 Ec^2}{8R}$

$$P_t' = \frac{m_a^2 Ec^2}{8R} + \frac{m_a^2 Ec^2}{8R}$$

$$= \frac{m_a^2 Ec^2}{4R} \qquad = \frac{m_a^2 Pc}{2}$$

Therefore the power saving with respect to AM is

$$\text{Power Saving} = \frac{P_t - P_t'}{P_t} \times 100$$

$$= \frac{[1 + m_a^2 / 2] P_C - [m_a^2 / 2] P_C}{[1 + m_a^2 / 2] P_C}$$

$$= \frac{P_C}{[1 + m_a^2 / 2] P_C}$$

$$\% \text{ Power Saving} = \frac{2}{2 + m_a^2} \times 100$$

If $m_a = 1$, then power saving (2/3) x 100 = 66.7 %

66.7% of power is saved by using DSB-SC-AM.

Advantages

DSB-SC is more efficient in transmitted power as compared to DSB-FC.

DSB-FC hs better signal to noise ratio as compared to SSB transmission.

Disadvantages

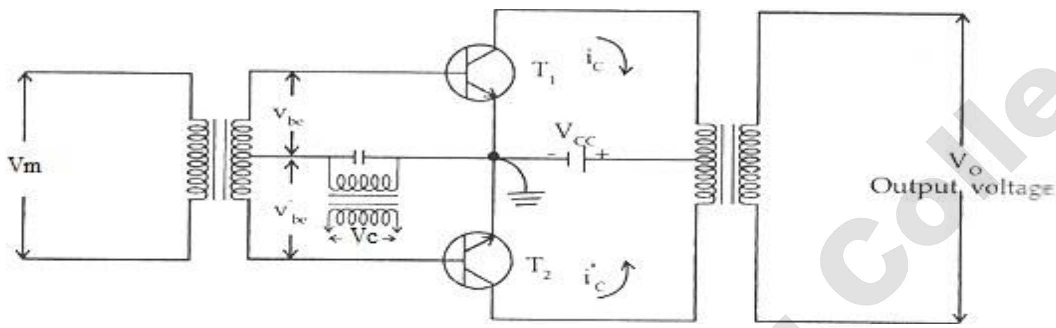
Bandwidth remains same as DSB-FC even though the carrier is suppressed.

GENERATION OF AM-DSBSC

1. Balanced Modulator Method
2. Ring Modulator Method

BALANCE MODULATOR METHOD

- The same circuit can be used to generate AM with carrier. The main difference between AM with carrier generation and DSB-SC –AM is the feeding points of the carrier and modulating signals are interchanged.
- The transistor is operated in a balanced mode thus heavy filtering is not required to remove the unwanted harmonics.
- It is assumed that the two transistors are identical and the circuit is symmetrical. The operation is confined in the nonlinear region of the active devices employed in this circuit. The carrier voltages across the upper and lower part of the secondary windings of the center tap transformers are equal in magnitude and opposite in phase.



BALANCED MODULATOR

Principle of operation:

- The modulating signal is applied as the input to the transistor T_1 and T_2 and the carrier signal is applied to the common input of the push pull amplifier configuration. Because of centre tap transformers are equal and opposite in phase $V_m = -V_m$
- Input to the transistor T1 is given by,

  $V_{be} = e_m(t) + e_c(t)$

  $V_{be} = E_m \sin\omega_m t + E_c \sin\omega_c t$ -- (1)
- Similarly Input to the transistor T2,

  $V'_{be} = - e_m(t) + e_c(t)$

  $V'_{be} = - E_m \sin\omega_m t + E_c \sin\omega_c t$ -- (2)

By using the non-linearity property the collector current can be written as per square law equation

$i_c = a_1 V_{be} + a_2 V_{be}^2 + \ldots\ldots$ --- (3)

$i'_c = a_1 V'_{be} + a_2 V'^2_{be} + \ldots\ldots$ --- (4)

On substituting eqn 3 and 4 in 1 and 2 we get

$i_c = a_1 [E_m \sin\omega_m t + E_c \sin\omega_c t] + a_2 [E_m \sin\omega_m t + E_c \sin\omega_c t]^2$

$i_c = a_1 [E_m \sin\omega_m t + E_c \sin\omega_c t] + a_2 E^2_m \sin^2\omega_m t + a_2 E_c \sin^2\omega_c t + 2a_2 E_m E_c \sin\omega_m t \sin\omega_c t$ ------ (5)

Similarly

$i_c = a_1 [-E_m \sin\omega_m t + E_c \sin\omega_c t] + a_2 [-E_m \sin\omega_m t + E_c \sin\omega_c t]^2$

$i_c = a_1 [-E_m \sin\omega_m t + E_c \sin\omega_c t] + a_2 E^2_m \sin^2\omega_m t + a_2 E^2_c \cos^2\omega_c t - 2a_2 E_m E_c \sin\omega_m t \sin\omega_c t$ ----------- (6)

The output AM voltage V_0 is given by $V_0 = K[i_c - i'_c]$ --- (7)

Because i_c, i'_c flows in the opposite direction and 'K' is a constant depending on impedance or other circuit parameters.

On substituting equation 5 and 6 in 7 we get,

$$V_o = K[\, i_c - i'_c \,]$$

$$V_o = K\,[2a_1\, Em\, \sin\omega_m t + 4a_2\, Em\, Ec\, \sin\omega_m t\, \sin\omega_c t]$$

The output contains the original modulating signal and the two sidebands. The modulating signal has been suppressed by tuning the tank circuit to the center frequency $\pm\omega_c$

$$V_o = 4Ka_2\, Em\, Ec\, \sin\omega_m t\, \sin\omega_c t$$
$$= 2Ka_2\, Em\, Ec[\, \cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

- So only the side bands are present in the o/p. Hence the DSB-SC-AM generated.
- The main advantage of this modulator is saving power and efficiency because of the suppressing of the carrier.

<u>RING MODULATOR METHOD</u>

- The balanced Ring modulator circuit is widely used in carrier telephony suppresses both unwanted modulating and carrier signal in its output.
- Ring modulator is a type of product modulator which is used to generate DSB-SC Signal.
- The band pass filter is not used at the output hence the harmonic frequencies are automatically controlled.
- In a ring modulator circuit four diode are connected in the form of ring in which all the four diodes are connected in the same manner and are controlled by a square wave carrier signal $e_c(t)$.
- The carrier signal acts as a switching signal to alternate the polarity of the modulating signal at the carrier frequency.
- When no modulating signal is present diode D_1 and D_2 or D_3 and D_4 will conduct depending upon polarity of the carrier.

POSITIVE HALF CYCLE OF CARRIER:

- Diodes D_1 and D_3 are forward biased. At this time D_2 and D_4 are reverse biased and act like open circuits. The current divides equally in the upper and lower portion of the primary winding T2.
- The current in the upper part of the winding produces a magnetic field that is equal and opposite to the magnetic field produced by the current in the lower half of the secondary.
- Therefore the magnetic fields cancel each other out and no output is induced in the secondary. Thus the carrier is effectively suppressed.

NEGATIVE HALF CYCLE OF CARRIER:

- When the polarity of the carrier reverses. Diodes D_1 and D_2 are reverse biased and the diodes D_3 and D_4 will conduct. Again the current flows in the secondary winding of T1 and the primary winding of T2.
- The equal and opposite magnetic fields produced in T2 cancel each other and thus result in zero carrier output. The carrier is effectively balanced out.

PRINCIPLE OF OPERATION

- When both the carrier and the modulating signals are present, during positive half cycle of the carrier diodes D_1 and D_2 conduct, while diodes D_3 and D_4 does not conduct.
- During negative half cycle of the carrier voltage diodes D_3 and D_4 conduct and D_1 and D_2 does not conduct.
- When polarity of the modulating signal changes the result is a 180 phase reversal. At the time D_3 and D_4 are in forward bias.

  Let us consider the modulating voltage,

  $e_m(t) = E_m \sin\omega_m t$ -- (1)

  $e_c(t) = E_c \sin \omega_c t$ -- (2)

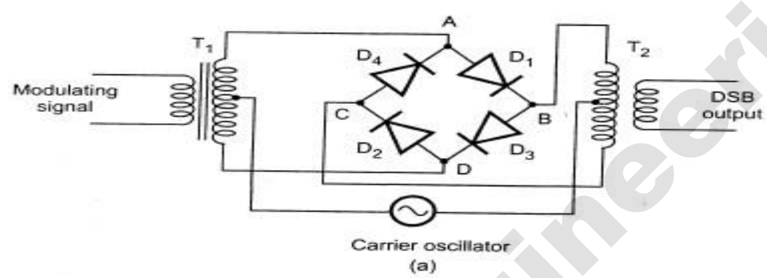
  The output voltage is $V_0(t) = e_m(t).e_c(t)$

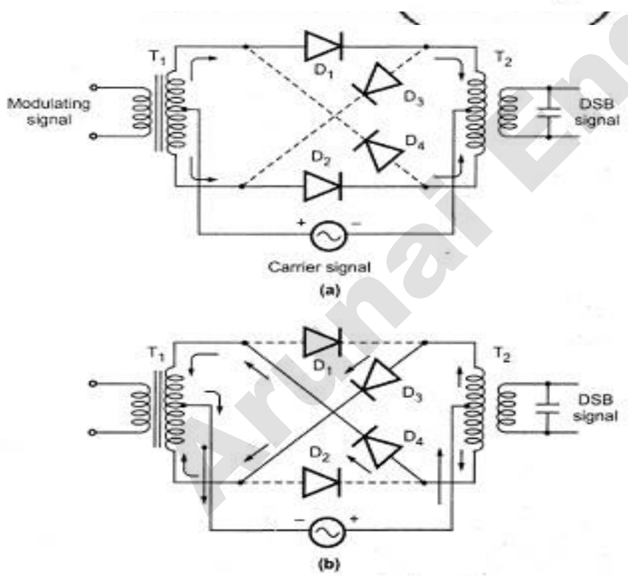
  $$=E_m.E_c \sin\omega_m t \sin\omega_c t$$

  $$V_0(t) = \frac{E_m.E_c}{2}\,[\,\underbrace{\cos(\omega_c - \omega_m)t}_{LSB} - \underbrace{\cos(\omega_c + \omega_m)t}_{USB}\,] \text{ -------------------- (3)}$$

The equation 3 shows that the o/p is free from the carrier and other higher order terms and it contains upper and lower sidebands only. The ring modulator circuit is also known as double balanced modulator because it is balanced with respect to both the baseband signal and the square wave carrier. The main advantage is the o/p is stable and long life, no external source to activate the diodes.

Circuit diagram: Waveform:





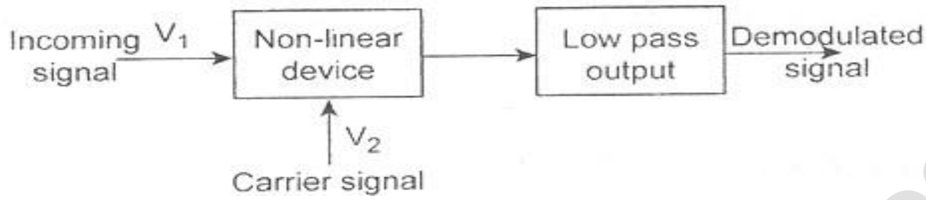
(a) Equivalent circuit of balanced modulator showing positive half cycle of carrier
(b) Equivalent circuit of balanced modulator showing negative half cycle of carrier

DEMODULATION/DETECTION OF AM-DSBSC

- SYNCHRONOUS OR COHERENT DETECTOR
- COSTAS PLL DETECTOR

SYNCHRONOUS OR COHERENT DETECTOR

- The coherent detector uses exact carrier synchronization for retrieving the message signal from modulated signal. These types of detectors are mainly used for detecting DSB&SSB signals.
- It consists of a product modulator with a low pass filter.
- For detecting signal local oscillator at the receiver end is required. The frequency and phase of the locally generated carrier and transmitter carrier must be synchronized that is exactly coherent.
- All types of linear modulation can be detected by using synchronous detector. It consists of a product modulator with LPF.



- The incoming signal is first multiplied with locally generated carrier and then passed through low pass filter. The filter bandwidth is same as the message bandwidth ω_m
- Tthe local oscillator should be exactly synchronized with carrier signal in both phase and velocity.
- **Considering the case of DSB-SC signal, the signal I/p is**

$$e_1(t) = E_m.E_c \sin\omega_m t \sin\omega_c t$$
$$e_2(t) = E \sin\omega_c t$$

The output of the non linear device which acts as a multiplying device is given by

$$e(t) = e_1(t).\ e_2(t) = E_m.E_c\, E \sin\omega_m t \sin^2\omega_c t$$

$$e(t) = E_m.E_c\, E\sin\omega_m t \left(\frac{1-\cos 2\omega_c t}{2}\right)$$

After passing through the LPF the second order harmonic term is eliminated.

$$e(t) = \frac{E_m.E_c\, E\sin\omega_m t}{2}$$

Assume the local oscillator carrier to have a phase difference with the transmitted carrier. The carrier is represented by

$$e_2(t) = E\sin(\omega_c t + \varphi)$$

The output of the non linear device is given by

$$e(t) = e_1(t).\ e_2(t) = E_m.E_c\, E \sin\omega_m t \sin\omega_c t \sin(\omega_c t + \varphi)$$

$$= E_m.E_c\, E \sin\omega_m t \sin\omega_c t\, [\sin\omega_c t\cos\varphi + \cos\omega_c t\sin\varphi]$$

$$= E_m.E_c\, E \sin\omega_m t \sin^2\omega_c t \cos\varphi + E_m.E_c\, E \sin\omega_m t \sin\omega_c t \cos\omega_c t \sin\varphi$$

$$= E_m.E_c\, E \sin\omega_m t \cos\varphi \left(\frac{1-\cos 2\omega_c t}{2}\right) + E_m.E_c\, E \sin\omega_m t \sin\omega_c t \cos\omega_c t \sin\varphi$$

$$= \frac{E_m.E_c\, E \sin\omega_m t \cos\varphi}{2} - \frac{E_m.E_c E \sin\omega_m t \cos\varphi\cos 2\omega_c t}{2} + E_m.E_c\, E \sin\omega_m t \sin\omega_c t \cos\omega_c t \sin\varphi$$

After passing through the LPF the second order harmonic term is eliminated.

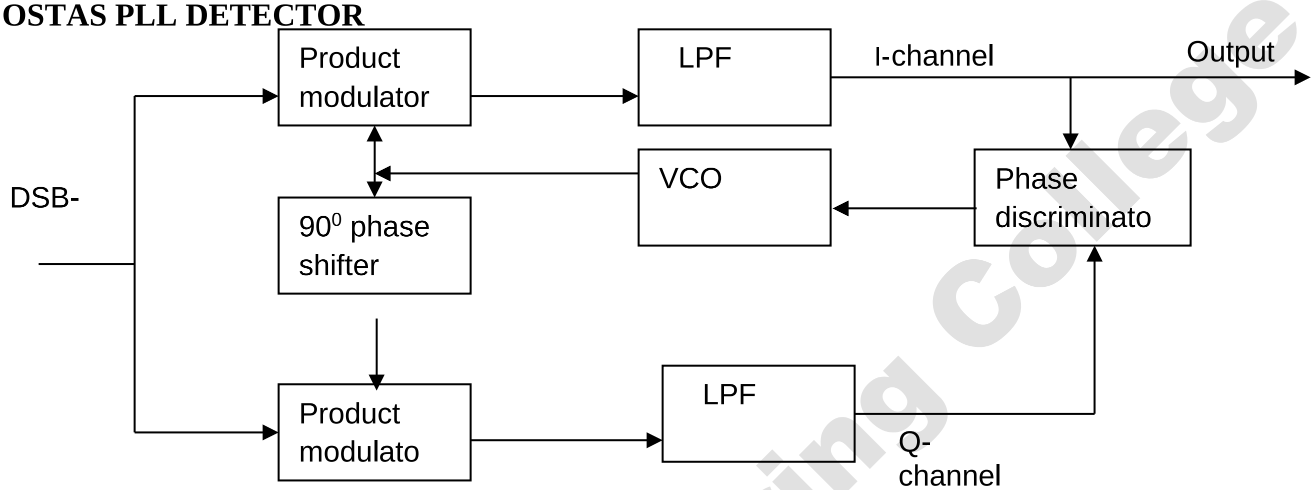
$$e(t) = \frac{E_m.E_c\, E\sin\omega_m t \cos\varphi}{2}$$

If $\varphi = 0$ then e(t) is maximum $e(t) = \underline{E_m.E_c\, E\sin\omega_m t}$

If φ = 90 then e(t) is minimum e(t) = 0

The demodulated signal $V_o(t)$ is therefore proportional to the message signal when the phase error, $\varphi = 0$ and it is minimum (zero) when $\varphi = \pm\frac{\pi}{2}$. Thus the phase error φ in the local oscillator causes the detector output to be attenuated by a factor equal to $cos\varphi$. As long as the phase error is constant, the detector provides an undistorted version of the original baseband signal.

COSTAS PLL DETECTOR



- Costas receiver is one of the method for obtaining a practical synchronous receiver suitable for demodulating DSB-SC waves. It consists of two coherent detectors supplied with the same input signal.
- One detector is supplied with the DSB-SC AM and locally generated carrier which is in phase with the transmitted carrier. This detector is known as **"In-phase coherent detector or I channel".**
- The other detector is supplied with the DSB-SC AM and locally generated carrier which is quadrature phase with the transmitted carrier. This detector is known as **"Quadrature coherent detector or Q channel".**
- These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.

Operation of the circuit:

- In this case I channel output contains the desired demodulated signal where as Q channel output is zero due to the quadrature null effect of Q channel.
- Suppose there is some phase shift φ radians between local oscillator carrier and the transmitting carrier then I channel output will remain essentially unchanged. But Q channel output contains some signal which is proportional to sin φ
- This Q channel output will have same polarity as the I channel output for one direction of local oscillator whereas the polarity will be opposite to the I channel for the other direction of phase shift.
- Thus the I and Q channel outputs are combined in phase discriminator
- The phase discriminator provides a d.c. control signal which may be used to correct local oscillator phase error.
- The local oscillator is a voltage controlled oscillator. Its frequency can be adjusted by an error control d.c signal.
- The costas receiver ceases phase control when there is no modulation and that phase lock has to be re-established with reappearance of modulation.

SINGLE SIDEBAND SUPPRESSED CARRIER

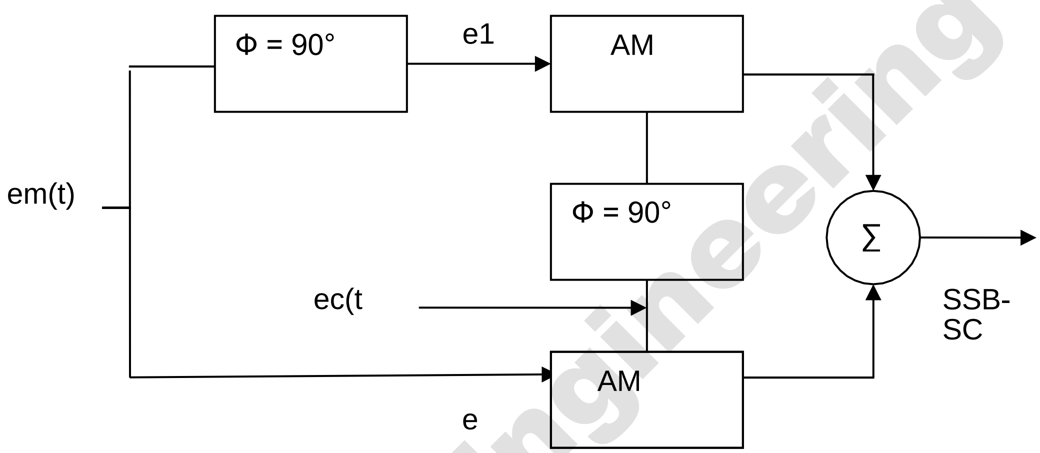
In AM with carrier both the transmitting power and bandwidth is wasted. Hence the DSB-SC AM scheme has been introduced in which power is saved by suppressing the carrier component but the bandwidth remains same.

- Increase in the saving of power is possible by eliminating one sideband in addition to the carrier component because the USB and LSB are uniquely related by symmetry about the carrier frequency. So either one sideband is enough for transmitting as well as recovering the useful message. The block diagram of SSB-SC AM is shown in figure.

- As for as transmission information is concerned only one side band is necessary. So if the carrier and one of the two sidebands are suppressed at the transmitter, no information is lost.

- This type of modulation is called as single side band suppressed carrier-AM and the SSB system reduces the band width by half.

Block diagram of SSB-SC AM is shown in figure.



- The single side band suppressed carrier can be obtained as follows,
- In order to suppress one of the sideband, the input signal fed to the modulator1 is 90° out of phase with that of the signal fed to the modulator'2'.

Let $e_1(t) = E_m \cos\omega_m t \,.E_c \cos \omega_c t$

$e_2 (t) = E_m \sin\omega_m t \,.E_c \sin\omega_c t$

Therefore, $e(t)_{SSB} = e_1(t) + e_2(t)$

$= E_m E_c [\cos\omega_m t.\ \cos\omega_c t + \sin \omega_m t.\ \sin\omega_c t]$

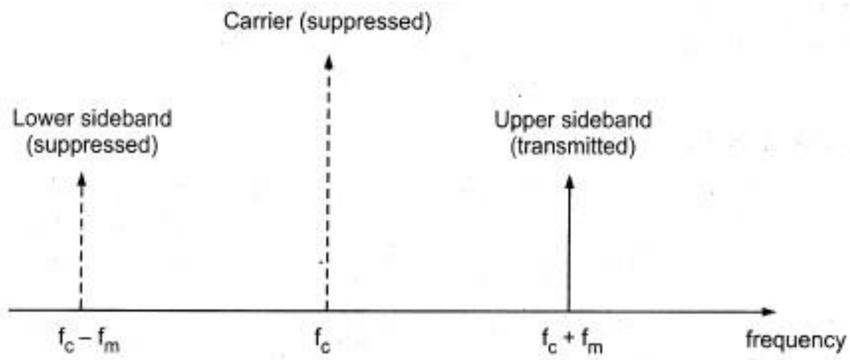
$e(t)_{SSB} = E_m E_c \cos (\omega_c - \omega_m) t.$

We know that for DSB-SC AM

$$e_{DSB\text{-}SC} (t) = \frac{E_m E_c}{2} [\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

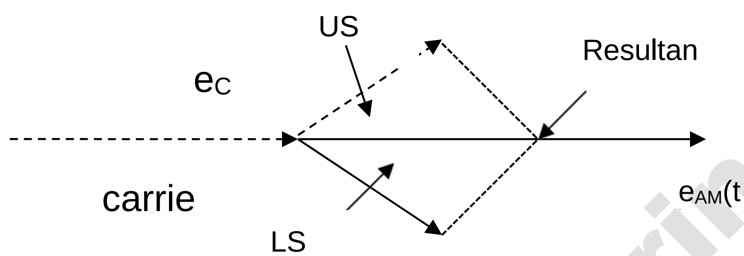
When comparing equations of $e(t)_{SSB\text{-}SC}$ and $e_{DSB\text{-}SC} (t)$, one of the sideband is suppressed. Hence this scheme is known as SSB-SC AM.

Frequency Spectrum Of SSB-SC-AM:



- The Frequency spectrum shows that only one side band signal is present , the carrier and the other sideband signal are suppressed. Thus the bandwidth required reduces from 2 ω_m to ω_m i.e., bandwidth requirement is reduced to half compared to AM & DSB-SC signals.

Phasor representation of SSB-SC-AM:



Power calculation:

Power in sidebands $P_t'' = P_{LSB} = P_{USB} = m_a^2 V_c^2/8R = m_a^2 Pc / 4$

Power saving with respect to AM with carrier

$$\text{Power saving} = \frac{P_t - P_t''}{P_t} \times 100$$

where P_t = Total power transmitted.

$$\text{Power saving} = \frac{[1+ m_a^2/2] P_C - [m_a^2/4] P_C}{[1+ m_a^2/2] P_C} = \frac{P_C + [m_a^2/2] P_C - [m_a^2/4] P_C}{[1+ m_a^2/2] P_C}$$

$$\frac{1 + \frac{m_a^2}{4}}{1 + \frac{m_a^2}{2}} \times 100$$

$$= \frac{\frac{4 + m_a^2}{4}}{\frac{2 + m_a^2}{2}} \times 100$$

$$= \frac{4 + m_a^2}{4 + 2m_a^2} \times 100$$

If m_a=1 then %power saving= (5/6) x100=83.3%

So we can save 83.3% of power with respect to AM with carrier.

We can also find the power in SSB-SC-AM with respect to AM with DSB-SC system,

$$powersaving = \frac{p_t' - p_t''}{p_t'} \times 100$$

$$= \frac{\frac{m_a^2 P_c}{2} - \frac{m_a^2 P_c}{4}}{\frac{m_a^2 P_c}{2}} \times 100$$

If m_a=1 then %power saving=50%.therefore 50%power is saved with respect to DSB-SC System.

Applications of SSB

1. Used to save applications where such a power saving is warranted, i.e., in mobile system, in which weight and power consumption must naturally be kept low.
2. Single sideband modulation is at a premium. Point-to-Point communication, land, air, maritime mobile communication, TV, Telemetry, Military and Radio navigation are the greatest use of SSB in one form or another.

SSB Advantages

- ***Power conservation***: Much less total transmitted power required to produce the same quality signal.
- ***Bandwidth conservation***: Half of the bandwidth of conventional AM bandwidth.
- ***Selective Fading***: Not present in SSBSC.
- ***Noise Reduction:*** Since SSB uses half the bandwidth, the thermal noise power is reduced to half. Hence immunity to selective fading is improved.

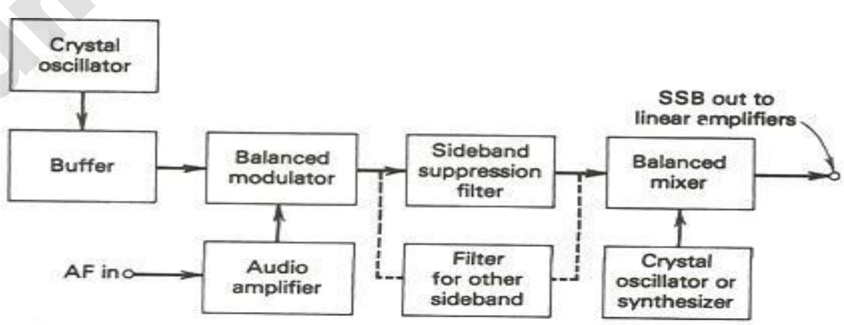
SSB Disadvantages

- ***Complex Receivers***: Required carrier recovery and synchronization circuit adds cost, complexity and size.
- ***Tuning Difficulties***: Complex and Expensive Tuning Circuits.

GENERATION OF SSB

1. Filter Method
2. Phase Shift Method
3. Modified Phase Shift Method or Weaver Method

FILTER METHOD



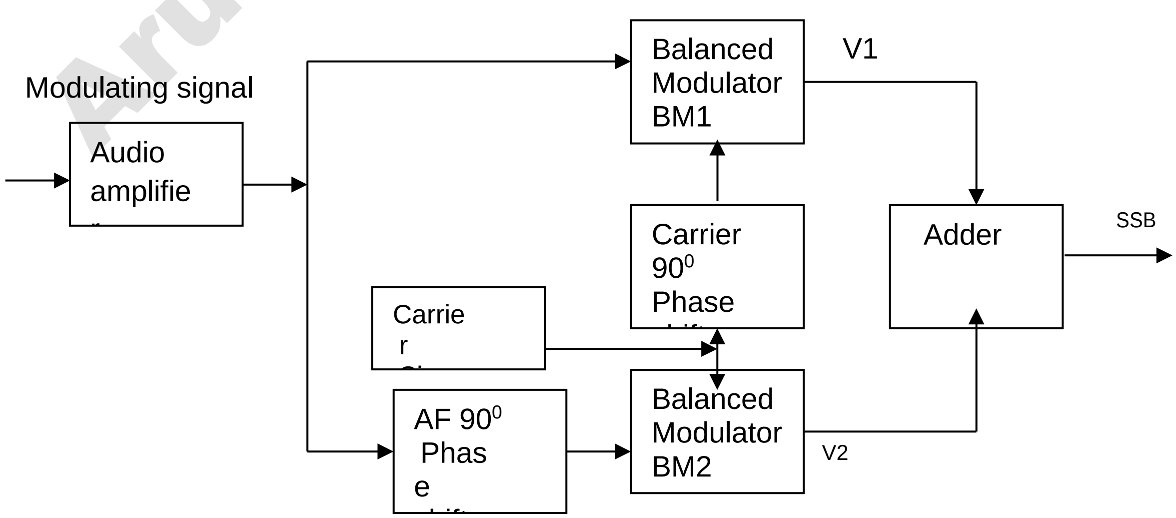
In this method of SSB generation, after the BM, the unwanted sideband is removed (actually heavily attenuated) by a filter and hence this name. The filter may be LC, ceramic or mechanical depending upon the carrier frequency and other requirements. Such a filter must have a flat bandpass and extremely high attenuation outside the passband.

In radio communication system, the frequency range used for voice is 300 Hz to about 2800 Hz in most cases. If it is required to suppress the lower sideband and if the transmitting frequency is f_c, then the lowest frequency that this filter must pass without attenuation is f_c+300 Hz whereas the highest frequency that must be fully attenuated is f_c-300 Hz. So we need a filter whose transition band is very low. This situation becomes worse if lower modulating frequencies are employed, such as the 50 Hz minimum in AM broadcasting. In order to obtain a filter response curve with skirts as steep the ‘Q’ of the tuned circuits must be very high.

The initial modulation takes place in the balanced modulator at a low frequency (such as 100 kHz) because of the difficulty of making adequate filters at higher frequencies. The filter is a BPF with a sharp cutoff frequency at either side of the bandpass to obtain satisfactory adjacent sideband rejection. The filtered signal is up-converted in a mixer to the final transmitter frequency and then amplified before being coupled to the antenna. The integrated ceramic filters are used as sideband filters. The drawback of filter method is that it requires sharp filtering, which requires filters with high Q. Primary modulation cannot be done at the transmitting frequency which is another drawback of the filter method.

PHASE SHIFT METHOD

- This method avoids the prime disadvantage of filtering method. That is requirement of a sideband filter with a narrow transition band and it cannot be used for very low and very high frequencies.
- This method does not have any sideband filters and the primary modulation can be done at the transmitting frequency. The unwanted sideband can be removed by generating the components of sideband out of phase.
- If the undesired sideband is LSB then the two LSB are generated such that they are 180 out of phase with each other. So that USB add with each other and LSB cancel each other. When two undesired sideband components are added they cancel each other with only the presence of desired signal.
- Two balanced modulators and two phase shifters are used. One of the modulator BBM1 receives the carrier voltage shifted by 90^0 and the modulating voltage, where as another balanced modulator BBM2 receives the modulating voltage shifted by 90^0 and the carrier voltage.
- The carrier signal is cancelled out by both the balanced modulator and then unwanted sidebands cancel at the output of the summing amplifiers and hence produces SSB signal.



PHASE SHIFT METHOD

Mathematical analysis:

For modulator1 (BM1)

$$e_c(t) = E_c \sin(\omega_c t+90^0) = E_c \cos\omega_c t$$
$$e_m(t)= E_m \sin\omega_m t$$

For modulator2(BM2)

$$e_c(t) = E_c \sin\omega_c t$$
$$e_m(t)= E_m \sin(\omega_m t+ 90^0) = E_m \cos\omega_m t$$

Output for modulator(BM1)isV_1

$$V_1=E_c \cos\omega_c t\ E_m \sin\omega_m t \text{ --------------------------------- (1)}$$

Output for modulator(BM2)isV_2

$$V_2=E_c \sin\omega_c t.\ E_m \cos\omega_m t \text{ -------------------------------- (2)}$$

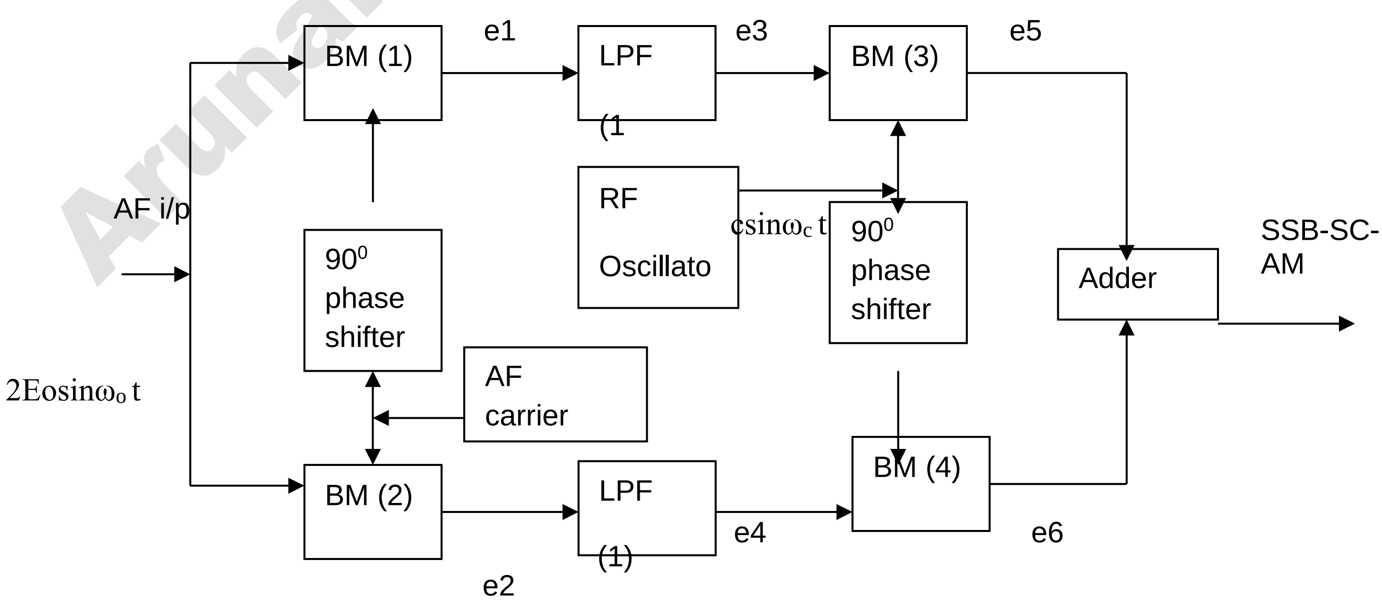
The output of adder is

$$V_0 = V_1 + V_2$$
$$V_0= Ec\ Em \cos\omega_c t \sin\omega_m t + Ec\ Em \sin\omega_c t\cos\omega_m t$$
$$=Ec\ Em \sin(\omega_c + \omega_m)t \text{------------------------------- (3)}$$

Thus one of the side band is cancelled where as the other is reinforced. This method avoids the use of filters.

MODIFIED PHASE SHIFT METHOD (OR) WEAVER'S METHOD

- The modified phase shift method overcomes the limitation of phase shift method. That is AF phase shift network is required to operate over a large range of audio frequencies but also retains the advantage like its ability to generate SSB at any frequency and use of low audio frequency.
- This method provides both RF and AF oscillator phase shift and also used in low frequency and so it can be used for both audio and radio frequencies.



WEAVER'S METHOD

- Modulators 1 and 2 both have the unshifted modulating signal as inputs. BM1 takes low frequency subcarrier with a 90 phase shift from the AF oscillator. BM2 receives the subcarrier signal directly from the oscillator.
- This method tries to aEoid the phase shift of audio frequencies and combine the audio frequency carrier with AF which lies in the middle of audio frequency.
- The low pass filter at the output of BM1 and BM2 with cut off frequency ensures the input to the balance modulator BM3 and BM4. The output of BM3 and BM4 gives the desired sideband suppression.

MATHEMATICAL ANALYSIS

The modulating signal is $em = Em \sin\omega_m t$ ----------------------------(1)

The A.F carrier(sub carrier signal) $eo(t) = 2E_0 \sin\omega_0 t$ -------------------------(2)

The R.F carrier $ec(t) = 2Ec \sin\omega_c t$ ---------------------------(3)

The input for Balanced modulator 1(BM1) is

$$em(t) = Em \sin\omega_m t$$
$$eo(t) = 2E_0 \sin(\omega_0 t + 90^0)$$

The output for Balanced modulator 1(BM1) is

$$e1 = Em \sin\omega_m t \; 2E_0 \sin(\omega_0 t + 90^0)$$
$$= Em\, E_0 [\cos(\omega_0 t + 90^0 - \omega_m t) - \cos(\omega_0 t + 90^0 + \omega_m t)] \text{---------} (4)$$

The input for Balanced modulator 2(BM2) is

$$em(t) = Em \sin\omega_m t$$
$$eo(t) = 2E_0 \sin\omega_0 t$$

The output for Balanced modulator 2(BM2) is

$$e2 = Em \sin\omega_m t \; 2E_0 \sin\omega_0 t$$
$$= Em\, E_0 [\cos(\omega_0 t - \omega_m t) - \cos(\omega_0 t + \omega_m t)] \text{---------------------} (5)$$

The LPF1, 2 eliminates the upper sidebands of the modulator. Hence USB is suppressed

The Output of LPF1 is $e3 = Em\, E_0 [\cos(\omega_0 t + 90^0 - \omega_m t)$ --------------------(6)

The Output of LPF2 is $e4 = Em\, E_0 \cos(\omega_0 t - \omega_m t)$ ---------------------------(7)

The output of LPF1, 2 are given to BM3,BM4.

The input for Balanced modulator 3(BM3) is

$$e3 = Em\, E_0 [\cos(\omega_0 t + 90^0 - \omega_m t)$$
$$ec(t) = 2Ec \sin\omega_c t$$

The output for Balanced modulator 3(BM3) is

$$e5 = 2Em\, E_0\, Ec \cos(\omega_0 t + 90^0 - \omega_m t) \sin\omega_c t$$

Assume $Em = E_0 = Ec = 1$, then

$$e5 = 2\cos(\omega_0 t + 90^0 - \omega_m t) \sin\omega_c t$$

$=\sin[(\omega_c + \omega_o - \omega_m)t + 90^0] + \sin[(\omega_c - \omega_0 + \omega_m)t - 90^0]$ -------------(8)

The input for Balanced modulator 4(BM4) is

$e4 = Em\ E_0 \cos(\omega_0 t - \omega_m t)$

$ec(t) = 2Ec \sin(\omega_c t + 90^0)$

The output for Balanced modulator 4(BM4) is

$e6 = Em\ E_0 \cos(\omega_0 t - \omega_m t)\ 2Ec \sin(\omega_c t + 90^0)$

Assume $Em = E_0 = Ec = 1$, then

$e6 = 2\cos(\omega_0 t - \omega_m t) \sin(\omega_c t + 90^0)$

$= \sin[(\omega_c + \omega_o - \omega_m)t + 90^0] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90^0]$ ------------ (9)

The output of adder is $e_{SSB\text{-}SC}(t) = e5 + e6$

$= \sin[(\omega_c + \omega_o - \omega_m)t + 90^0] + \sin[(\omega_c - \omega_0 + \omega_m)t - 90^0] +$
$\sin[(\omega_c + \omega_o - \omega_m)t + 90^0] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90^0]$

$= 2\sin[(\omega_0 + \omega_c - \omega_m)t + 90^0]$

The other two terms cancel with each other because it is out of phase.

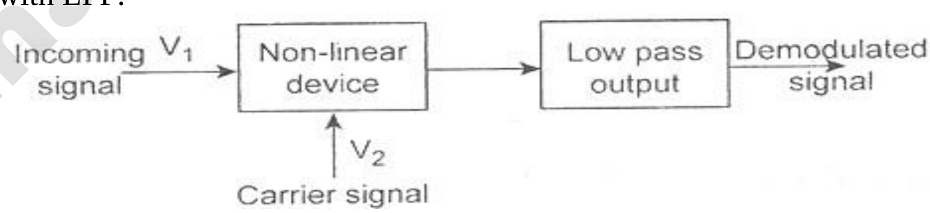
$$\mathbf{e_{SSB\text{-}SC}(t) = 2\cos[(\omega_0 + \omega_c - \omega_m)]t}$$

The final RF output frequency is $\boldsymbol{\omega_0 + \omega_c - \omega_m}$ which is essentially the lower side band of RF carrier $\boldsymbol{\omega_0 + \omega_c}$.

DEMODULATION/DETECTION OF AM-SSBSC

SYNCHRONOUS OR COHERENT DETECTOR

- The coherent detector uses exact carrier synchronization for retrieving the message signal from modulated signal. These types of detectors are mainly used for detecting DSB&SSB signals.
- It consists of a product modulator with a low pass filter.
- For detecting signal local oscillator at the receiver end is required. The frequency and phase of the locally generated carrier and transmitter carrier must be synchronized that is exactly coherent.
- All types of linear modulation can be detected by using synchronous detector. It consists of a product modulator with LPF.



- The incoming signal is first multiplied with locally generated carrier and then passed through low pass filter. The filter bandwidth is same as the message bandwidth ω_m
- Tthe local oscillator should be exactly synchronized with carrier signal in both phase and velocity.
- **Considering the case of SSB-SC signal, the signal I/p is**

$$e_1(t) = \frac{E_m . E_c \cos(\omega_c - \omega_m)t}{2}$$

$e_2(t) = E\sin\omega_c t$

The output of the non linear device which acts as a multiplying device is given by

$$e(t)= e_1(t).\ e_2(t)= \frac{E_m.E_c\ E \cos(\omega_c - \omega_m)t \sin\omega_c t}{2}$$

$$e(t)= \frac{E_m.E_c\ E}{2}\left(\frac{\sin\omega_m t\ + \sin\ (2\omega_c - \omega_m)t}{2}\right)$$

The first term of the output is the modulating frequency signal that is passed on to the output. The second component is a RF component and is attenuated by the filter. Thus the synchronous detector is capable of demodulating SSB-SC signal

$$e(t)= \frac{E_m.E_c\ E\sin\omega_m t}{4}$$

The synchronous detector is effective only when locally generated carrier is properly synchronize with the transmitter power. Assume the local oscillator carrier to have a phase difference with the transmitted carrier. The carrier is represented by

$$e_2(t)=E\sin(\ \omega_c t + \varphi\ )$$

The output of the non linear device is given by

$$e(t)= e_1(t).\ e_2(t)= \frac{E_m.E_c \cos(\omega_c - \omega_m)t}{2} E\sin(\ \omega_c t + \varphi\ )$$

$$= \frac{E_m.E_c\ E}{2} \cos(\omega_c - \omega_m)t \sin(\ \omega_c t + \varphi\ )$$

$$=\frac{E_m.E_c\ E}{2} (\ \cos\omega_c t \cos\omega_m t+\sin\omega_m t \sin \omega_c t)(\sin\omega_c t \cos\varphi + \cos\omega_c t \sin \varphi\ )$$

$$=\frac{E_m.E_c E}{2} [\ \cos\omega_c t \cos\omega_m t \sin\omega_c t \cos\varphi + \sin\omega_m t \sin^2 \omega_c t \cos\varphi +$$
$$\text{Cos}^2\omega_c t \cos\omega_m t \sin \varphi + \cos\omega_c t \sin \omega_c t \cos\omega_m t \cos\varphi\ ]$$

$$=\frac{E_m.E_c\ E}{2} [\ \cos\omega_c t \cos\omega_m t \sin\omega_c t \cos\varphi + \sin\omega_m t \cos\varphi \left(\frac{1-\cos 2\omega_c t}{2}\right)$$
$$\text{Cos}^2\omega_c t \cos\omega_m t \sin \varphi + \cos\omega_c t \sin \omega_c t \cos\omega_m t \cos\varphi\ ]$$

After passing through the LPF the second order harmonic term is eliminated.

$$e(t)= \frac{E_m.E_c\ E\sin\omega_m t \cos\varphi}{4}$$

If $\varphi = 0$ then e(t) is maximum $e(t) = \frac{E_m.E_c\ E\sin\omega_m t}{4}$

If $\varphi = 90$ then e(t) is minimum $e(t) = 0$

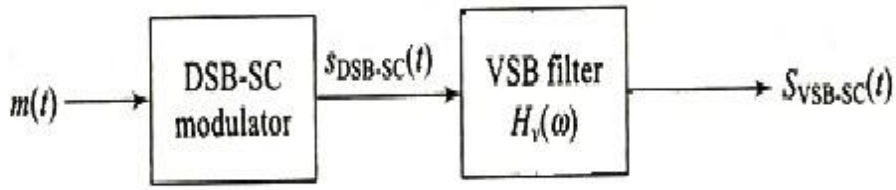
Thus there is a phase delay due to improper synchronization.

VESTIGEAL SIDE BAND MODULATION

Definition : One of the sideband is partially suppressed and vestige (portion) of the other sideband is transmitted, This vestige (portion) compensates the suppression of the sideband. It is called vestigial sideband transmission.

Generation and demodulation of VSB:

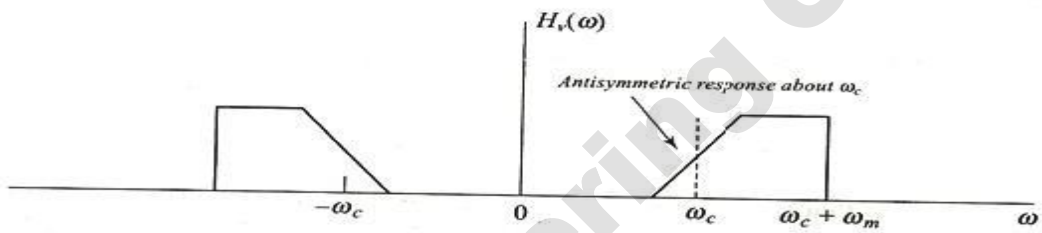
A VSB signal is obtained as shown figure below by suppressing one of the sidebands of a DSBSC using a VSB filter.



VSB filter is a BPF having an asymmetric frequency response in the transition band, positioned in such a way that the carrier frequency corresponds to the middle of the transition band. From the figure,

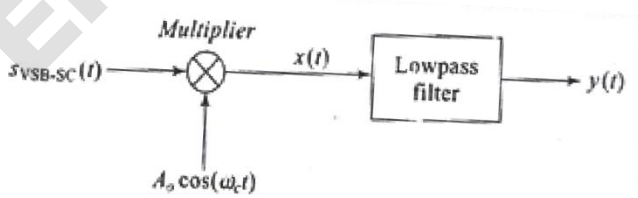
$$S_{DSB-SC}(t) = A_c m(t) cos(\omega_c t)$$

The transfer function of the VSB filter is $H_v(\omega)$.



VSB filter characteristics

The coherent detector is a sort of universal detector of AM signals in the sense that DSBSC, conventional AM and SSBSC can all be detected successfully by using it. It would be natural to expect that coherent detection to work for the VSB-SC signal too.



The output of the multiplier in the VSB demodulator in the figure is given by

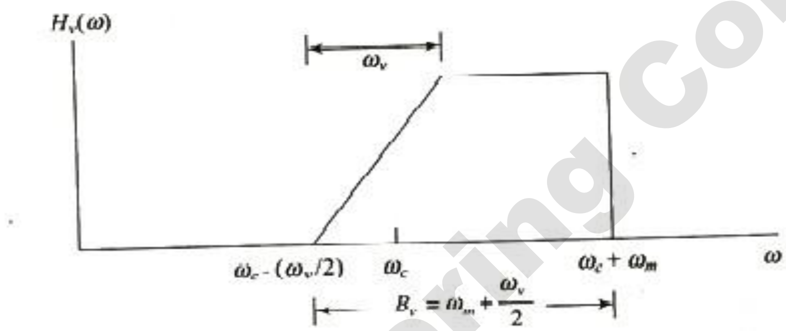
It is assumed perfect synchrony between the transmitter and demodulator carriers.

Taking FT on both sides of equation (3), we obtain, The signal is passed through an ideal filter (LPF) to obtain If the VSB modulation is successful, then should be proportional to the message signal,

It is observed from are high pass terms, since they represent translated by These terms are blocked by the LPF which yields an output signal with the FT given by

VSBSC modulation and demodulation is considered to be successful if, where *k* is proportionality constant. Hence, for perfect demodulation, the required condition

This is called *vestigial symmetry condition*.



Transmission bandwidth of a VSB-SC signal

It is seen that the VSB bandwidth exceeds the corresponding SSB bandwidth ω_m by $\omega_v/2$. Thus, the filter transition bandwidth ω_v is an important parameter which decides the VSB bandwidth. It is observed in filter design that spectral components that lie in the transition region suffer distortion causing phase shifts.

Magnitude Response of VSB Filter

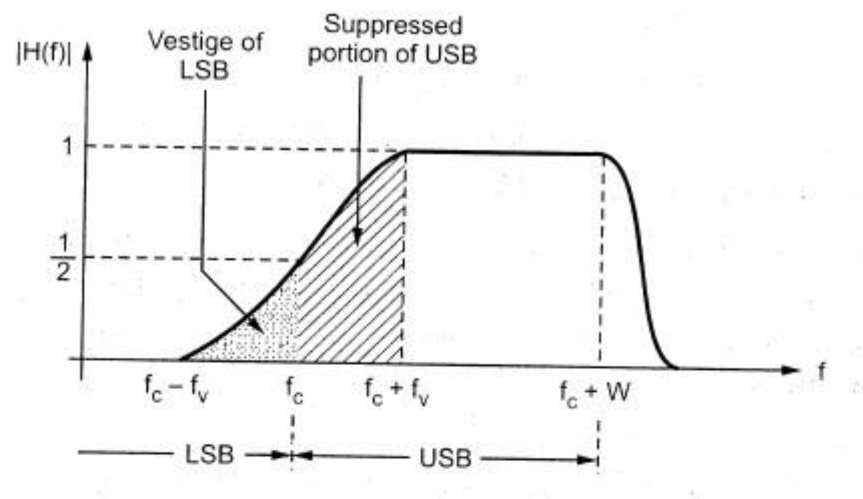
Fig. shows the magnitude response of VSB filter.

- Here observe that fc to fc+W is USB. It's portion from fc to fc +fv is suppressed partially. *fc* to fc - W is LSB. It's portion from fc -f*v* to fc is transmitted as vestige.
- Observe that H(fc)=1/2. And the frequency response fc-fv ≤ H(f)≤ fc+fv exhibits odd symmetry. The sum of any two frequency components in the range is

$$f_c - f_v \leq f \leq f_c + f_v$$

equal to unity. i.e H(f-fc) + H(f+fc) = 1

Phase response is linear



Spectrum of VSB filter

Advantages:
1. Low frequencies, near *fc* are, transmitted without any attenuation.
2. Bandwidth is reduced compared to DSB.

Applications:
VSB is mainly used for TV transmission, since low frequencies near *fc* represent significant picture details. They are unaffected due toVSB.

<u>UNIT -2</u>

Angle Modulation

Definition

We know that amplitude, frequency or phase of the carrier can be varied by the modulating signal. Amplitude is varied 'in AM. *When frequency or phase of* the carrier is varied by the modulating signal, then it is called angle modulation, There are two types of angle modulation.

1. Frequency Modulation :

When frequency of the carrier varies as per amplitude variations of modulating signal, then it is called Frequency Modulation (FM). Amplitude of the modulated carrier remains constant.

2. Phase Modulation :

When phase of the carrier varies as per amplitude variations of modulating signal, then it is called Phase Modulation (PM). Amplitude of the modulated carrier remains constant,

Frequency Modulation:

The frequency of the high frequency carrier signal is carried in accordance with the modulating signal.

$V_m = E_m \sin 2\pi fmt$

$V_c = E_c \sin 2\pi fct$

$V_{fm} = E_c \sin (2\pi fct + m_f \sin 2\pi fmt)$

$V_{fm} = E_c \sin (wct + m_f \sin wmt)$

$m_f \rightarrow$ modulating index of f_m

Relationship/Difference between FM and PM:

- The basic difference between FM and PM lies in which property of the carrier isdirectly varied by modulating signal. Note that when frequency of the carrier varies,phase of the carrier also varies and viceversa.
- But if frequency is varied directly, thenit .is called FM.,
- And if phase is varied. directly, then it is called PM.

The instantaneous phase deviation is denoted by θ *(t).* It is the instantaneous change in phase of the carrier with respect to reference phase. The instantaneous phase of the carrier is precise phase of the carrier at a given instant .It is mathematically expressed as,

Instantaneous phase =

$$\omega_c t + \theta(t) \quad ....(1)$$

Here θ(t) is the instantaneous phase deviation and ωc is the carrier frequency. Now the instantaneous frequency deviation is defined as

$$= \frac{d}{dt}\theta(t) = \theta'(t) \; Hz \quad ....(2)$$

Definition for instantaneous frequency deviation: It is the instantaneous change in carrier frequency. It is equal to the rate at which instantaneous phase deviation takes place.

Definition of instantaneous frequency:It is the frequency of the carrier t a given instant of time. It is given as

$$\text{Instantaneous frequency} = \omega_i(t) = \frac{d}{dt}\left[\omega_c t + \theta(t)\right]$$
$$= \omega_c + \theta'(t) \text{ rad/sec} \qquad \text{..................(3)}$$

Instantaneous phase deivation θ(t) is proportional to modulating signal voltage

$$\theta(t) = k\, e_m(t) \text{ rad} \qquad \text{-. ....(4)}$$

Where K is the deviation sensitivity of phase

Similarly the instatneous frequency deviation is proportional to modulating Signal

$$\theta'(t) = k_1\, e_m(t) \text{ rad/sec} \qquad \text{........................(5)}$$

Where k1 is the deviation sensitivity of frequency.

From equation (2), We have

$$\theta(t) = \int \theta'(t)\, dt$$
$$= \int k_1\, e_m(t)\, dt \qquad \text{.......................(6)}$$

Let the modulating signal be given as

$$e_m(t) = E_m \cos\omega_m t$$

Using the equation in equation (6)

$$\theta(t) = k_1 \int E_m \cos\omega_m t\, dt$$
$$= k_1 \frac{E_m}{\omega_m} \sin\omega_m t \qquad \text{.......................(7)}$$

The angle modulated wave is mathematically expressed as

$$e(t) = E_c \sin\left[\omega_c t + \theta(t)\right] \qquad \text{.......................(8)}$$

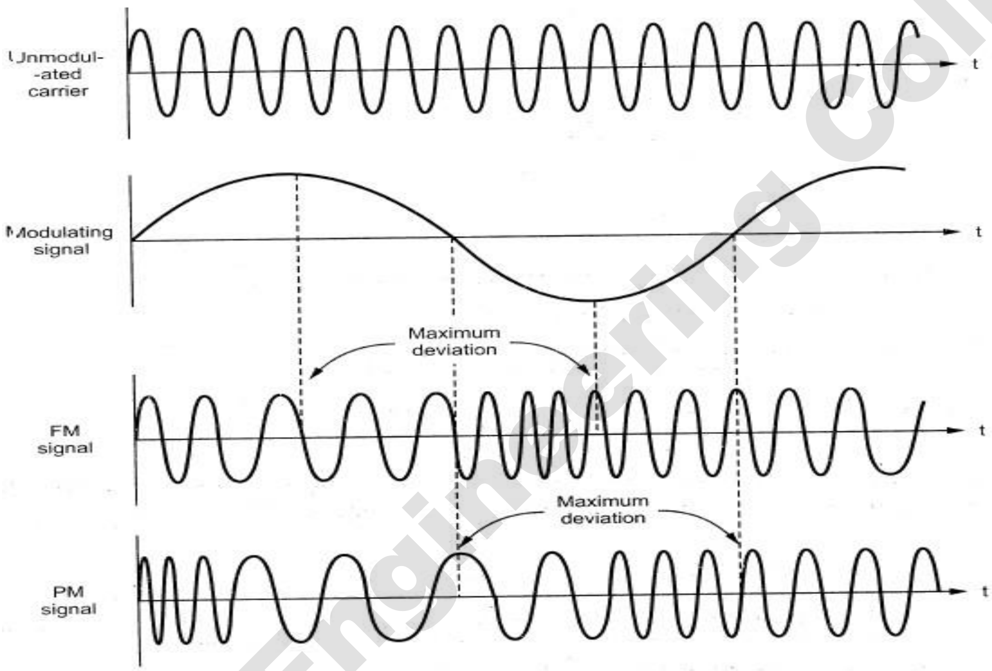
Using the value of θ(t) in the above equation from equation (7)

$$\text{FM equation : } \boxed{e(t) = E_c \sin\left[\omega_c t + \frac{k_1 E_m}{\omega_m} \sin \omega_m t\right]} \quad ....(9)$$

Similarly using the value of θ(t) from equation (5) in equation (8) we get

$$\boxed{\text{PM equation : } e(t) = E_c \sin\left[\omega_c t + k E_m \cos\omega_m t\right]} \quad ....(10)$$

FM and PM waveforms:



FM and PM waveforms

From the above waveform we can note the following

- For FM signal maximum frequency" deviation takes Place when Modulating signal is at positive .and negative peaks.
- For PM signal the maximum frequency. deviation takes place near zero crossings of the modulating signal.
- Both FM anid PM waveforms are identical except the phase shift.

Definition of Modulation index of PM and FM

The modulation index of PM is given as

$M = KE_m$

For FM It is the ratio of maximum frequency deviation (δ) to the modulating frequency (f_m).

$$m_f = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}} = \frac{\delta}{f_m}$$

- The maximum frequency deviation is the shift from centre frequency f_c when the amplitude of message is maximum.

$$\Delta f = K_1 E_m \text{ (Hz)}$$

K_1 = Deviation sensitivity.

The Bandwidth of FM:

- By Carson's rule the Bandwidth needed by fm is given as,

$$B\omega = 2(\delta + f_m max)$$

$\delta \rightarrow$ Maximum frequency Deviation

$f_m max$ is Maximum modulating frequency.

Deviation Ratio :

The modulation index corresponding to maximum modulating frequency is called deviation ratio.

$$\text{Deviation Ratio} = \frac{\text{Maximum frequency deviation}}{\text{Maximum modulating frequency}}$$

Frequency Spectrum of angle modualted wave :

- FM and PM analysis is quite complicated. It is derived with the help of Bessel function.

$$E_{fm} = E_c \sin(w_c t + mf \cos w_m t)$$

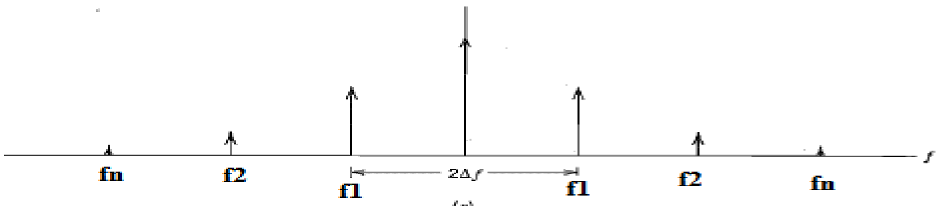
Using Bessel function this can be expanded as,

$$E_{fm} = A \{J_o mf \sin w_c t + J_1 mf [\sin(w_c + w_m)t - \sin(w_c - w_m)t]$$

$$+ J_2 mf [\sin(w_c + 2w_m)t - \sin(w_c - 2w_m)t] + J_3 mf [\sin(w_c + 3w_m)t$$

$$- \sin(w_c - 3w_m)t] + J_4 mf [\sin(w_c + 4w_m)t - \sin(w_c - 4w_m)t] \ldots\ldots\}$$

J_0, J_1, J_2, J_3 …. Are Bessel functions. The value of this depends on modulation index m_f.



From the figure the Bandwidth of FM is given by

$B = f_c + nf_m – f_c + nf_m$

$$BW = 2nf_m$$

Bessel Function Table :

m	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10
0	1	-	-	-	-	-	-	-	-	-	-
0.25	0.98	0.12	-	-	-	-	-	-	-	-	-
0.5	0.94	0.24	0.03	-	-	-	-	-	-	-	-
1	0.77	0.44	0.11	0.02	-	-	-	-	-	-	-
1.5	0.51	0.56	0.23	0.06	0.01	-	-	-	-	-	-
2	0.22	0.58	0.35	0.13	0.03	-	-	-	-	-	-
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	-	-	-	-	-
3	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	-	-	-	-
4	-0.04	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	-	-	-
5	0.18	-0.33	0.05	0.31	0.39	0.25	0.13	0.05	0.02	-	-

Classification of FM:

1. Narrowband FM
2. Wide band FM

Narrow band FM:

When the modulation index is less than I, it is called narrowband FM. The FM Equation given by eq. 9 can also be expressed as,

$$e(t) = E_c \cos\left[\omega_c t + \frac{k_1 E_m}{\omega_m} \sin \omega_m t\right]$$ **…………………(1)**

$$\frac{k_1 E_m}{\omega_m} = m,$$

$$e(t) = E_c \cos\left[2\pi f_c t + m \sin 2\pi f_m t\right]$$

Expanding the equation

$$e(t) = E_c \cos(2\pi f_c t) \cos[m \sin(2\pi f_m t)] - E_c \sin(2\pi f_c t) \sin[m \sin(2\pi f_m t)] \quad \text{.........(2)}$$

For narrowband FM, the modulation index, m is very small therefore following approximations can be considered.

$$\cos[m \sin(2\pi f_m t)] \approx 1$$

$$\sin[m \sin(2\pi f_m t)] \approx m \sin(2\pi f_m t)$$

Using this in equation (2)

$$e(t) = E_c \cos(2\pi f_c t) - m E_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

Expanding

$$e(t) = E_c \cos(2\pi f_c t) + \frac{1}{2} m E_c \left\{\cos 2\pi(f_c + f_m)t - \cos 2\pi(f_c - f_m)t\right\}$$

This equation gives the spectrum of narrowband FM. Observe that there is carrier frequency fc, upper sideband (fc + *fm)* and lower sideband (fc - *fm).*

Wide band FM

If the modulation index is higher than 10 it is called as wide band FM

$$\begin{aligned} e(t) &= E_c \cos\left[2\pi f_c t + m \sin 2\pi f_m t\right] \\ &= \mathrm{Re}\left[E_c\ e^{j(2\pi f_c t + m \sin 2\pi f_m t)}\right] \\ &= \mathrm{Re}\left[E_c e^{j 2\pi f_c t} \cdot e^{j m \sin 2\pi f_m t}\right] \end{aligned}$$

Let $x(t) = E_c e^{j m \sin 2\pi f_m t}$, then the above equation becomes

$$e(t) = \mathrm{Re}\left[x(t) e^{j 2\pi f_c t}\right] \quad \text{.........(1)}$$

Here $x(t) = E_c e^{j m \sin 2\pi f_m t}$ is periodic with fundamental frequency of f_m. It can be expressed with the help of Fourier series as,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n\ e^{j 2\pi n f_m t} \quad \text{........(2)}$$

where
$$C_n = f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} x(t)\, e^{-j\,2\pi n f_m t}\, dt$$

$$= f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} E_c\; e^{jm\sin\,2\pi f_m t}\; e^{-j\,2\pi n f_m t}\; dt$$

$$= f_m E_c \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} e^{j(m\sin\,2\pi f_m t - 2\pi n f_m t)}\, dt$$

put $y = 2\pi f_m t$ hence the limits will change from $-\pi$ to π.

$$\therefore \quad C_n = \frac{E_c}{2\pi}\int_{-\pi}^{\pi} e^{j(m\sin y - ny)} dy$$

The above integral is known as the nth order Bessel function of the first kind. It is given as

$$J_n(m) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(m\sin y - ny)} dy$$

$$C_n = J_n(m) E_c$$

Using the value of Cn in equation (2)

$$x(t) = \sum_{n=-\infty}^{\infty} J_n(m) E_c\, e^{j2\pi n f_m t}$$

Using the value x(t) in equation (1)

$$e(t) = \mathrm{Re}\left[\sum_{n=-\infty}^{\infty} J_n(m) E_c\; e^{j\,2\pi\, n f_m t}\,.\,e^{j\,2\pi f_c t}\right]$$

$$= E_c \sum_{n=-\infty}^{\infty} \mathrm{Re}\left[J_n(m) e^{j\,2\pi(f_c + n f_m)t}\right]$$

$$= E_c \sum_{n=-\infty}^{\infty} J_n(m)\cos\left[2\pi(f_c + n f_m)t\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The Fourier transform of the above equation becomes

$$E(f) = \frac{E_c}{2} \sum_{n=-\infty}^{\infty} J_n(m)\{\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)\}$$

This equation shows that there are infinite number of components located

fc±fm, fc±2fm,fc ±3fm…………….

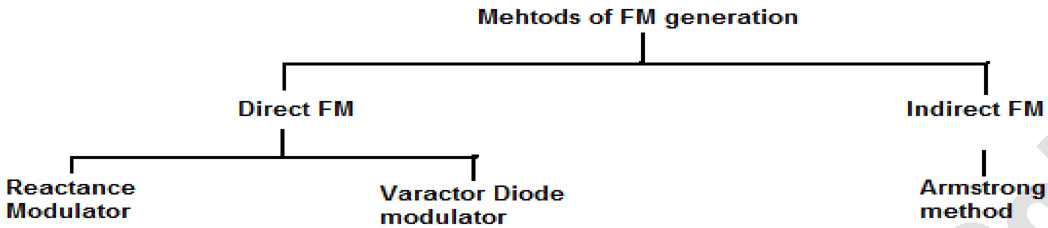
Comparison between FM and PM

	Frequency modulation	**Phase modulation**
1	The maximum frequency deviation depends upon amplitude of modulating voltage and modulating frequency	The maximum phase deviation depends only upon the amplitude of modulating voltage
2	Frequency of the carrier is modulated by modulating signal	Phase of the carrier is modulated by modulating signal
3	Modulation index is increased as modulation frequency is reduced and vice versa	Modulation index remains same if modulating frequency is changed
4	Noise immunity is bette than AM and PM	Noise immunity is better than AM but worse than FM

Generation of FM waves:

Fm Modulators: There are 2 types of FM modulators.

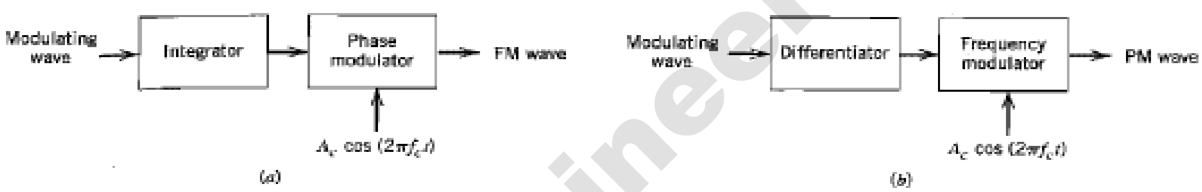
1. Direct Method
2. Indirect Method



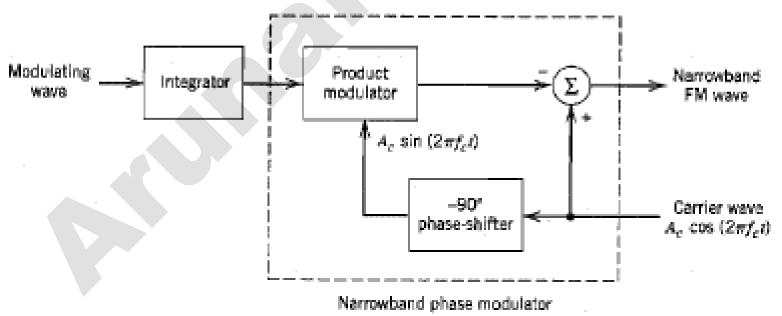
Direct FM Modulators : In this type the frequency of the carrier is varied directly by the modulating signal.

Indirect FM Modulators: In this type FM is obtained by phase modulation of the carrier.

Generation of Narrow band FM:

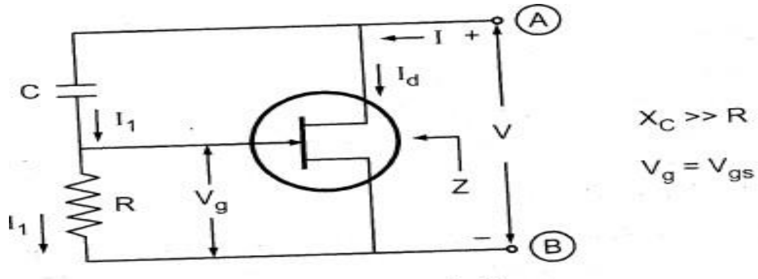


Ilustration of the relationship between FM and PM
Fig (a) Generation of FM using PM
Fig (b) Generation of PM from FM



Block diagram of a method for generating a narrowband FM signal.

Direct FM reactance modulator.



FET reactance modulator

- It behaves as reactance across terminal A-B.
- The terminal A-B of the circuit may be connected across the tuned circuit of the oscillator to get fm o/p.
- The varying voltage (modulating voltage) V, across the terminals A-B changes the reactance of FET.
- This charge in reactance can be inductive or capacitive.
- Neglecting the gate current, let the current through C & R be I_1.
- At the carrier freq. the reactance of C is much larger than

$$R \ \& \ I_1 = \frac{V}{R + 1/jwc}$$

Jwc >> R

$$I_1 = jwcV$$

From the Circuit,

$$V_g = I_1R = jwcrv$$
$$I_d = g_m v_g s = g_m V_g$$
$$I_d = jwcRg_m V$$

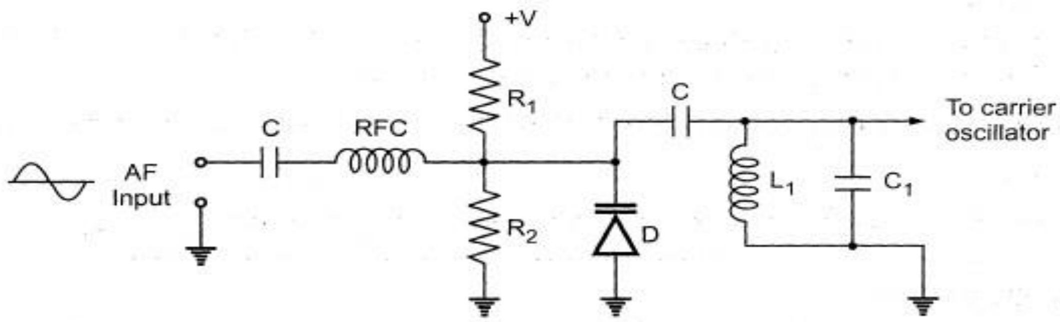
From the circuit impedance of the FET is,

$$Z = \frac{V}{Id}$$

$$= \frac{V}{jwCR\ gm\ V} = \frac{1}{jw\ (gmCR)} = \frac{1}{jw\ (C_{eq})}$$

- The impedance of FET is capacitive.
- By carrying the modulating voltage across FET, the operating paint g_m can be varied and hence C_{eq}.
- This change in the capacitance will change the frequency of the oscillator.

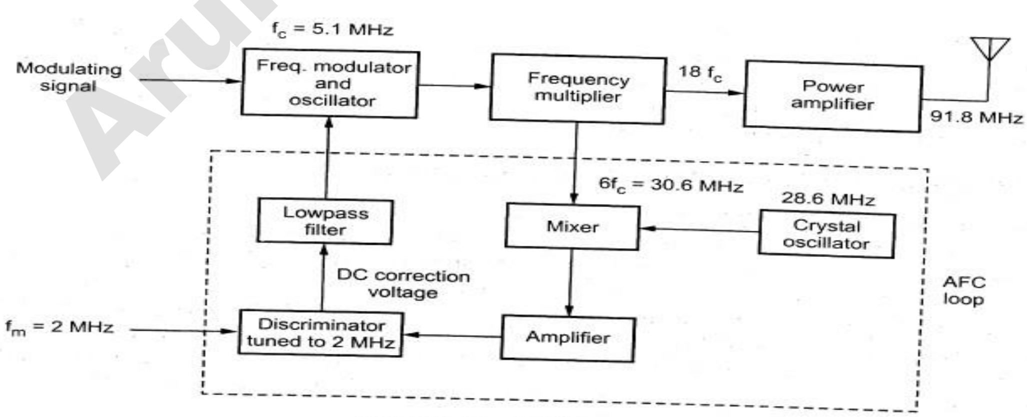
Frequency Modulation using Varactor diode.



Varactor diode for FM generation

- We know that the junction capacitance of the varactor diode changes as the reverse bias across it is varied.
- L_1 & C_1 forms the tank circuit of the carrier oscillator.
- The capacitance of the varactor diode depends on the fixed bias set by R_1 & R_2 & AF modulating signal.
- Either R1 or R2 is made variable.
- The radio frequency choke [RFC] has high reactance at the carrier frequency to prevent carrier signal from getting into the modulating signal.
- At +ve going modulating signal adds to the reverse bias applied to the varactor diode D, which decreases its capacitance & increases the carrier frequency.
- A –ve going modulating signal subtracts from the bias, increasing the capacitance, which decreases the carrier frequency.

Direct Fm Transmitters :



Crosby direct FM transmitter

- Fig. shows the FM Crosby transmitter with an AFC loop. (Automatic frequency correction loop).
- The Frequency modulator can be either a reactance modulator or voltage controlled oscillator.
- The carrier freq is 5.1MHz. which multiplies by 18 in three steps to produce a final frequency of 91.8 MHz.
- When the frequency modulated carrier is multiplied, its frequency & phase deviations are also multiplied.
- The rate at which the carrier is deviated is unaffected by the multiplication process. Hence the modulation index is multiplied.
- When an angle modulated carrier is heterodyned with another freq in a non linear mixer, the carrier can either be up converted or down converted.

AFC loop :

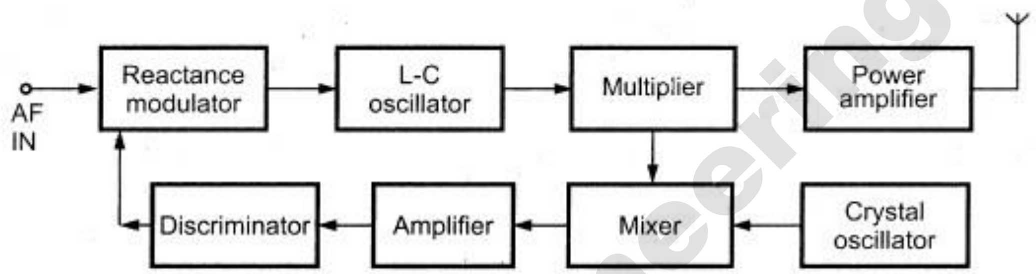
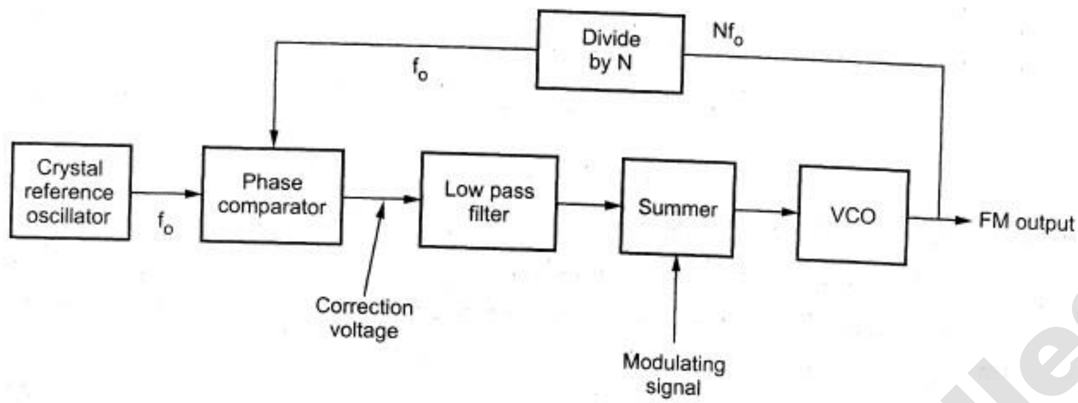


Fig. 2.3.1 Typical AFC circuit

- The purpose of the AFC loop is the achieve near crystal stability of the transmit carrier freq. without using a crystal in the carrier oscillator.
- The cassier frequency is mixed with a local oscillator freq and then down converted in freq. & the fed to a frequency discriminator.
- Frequency discriminator is a device whose o/p voltage is proportional to difference b/w i/p freq and its resonant freq.
- Discriminator responds to low freq changes in the carrier center freq because of master oscillator freq drift.
- When the discriminator responds to frequency deviation, the feedback loop would cancel the deviation and this remove the modulation.
- The dc correction voltage is added to the modulating signal to automatically adjust the master oscillator's centre frequency to compensate for low freq drift.

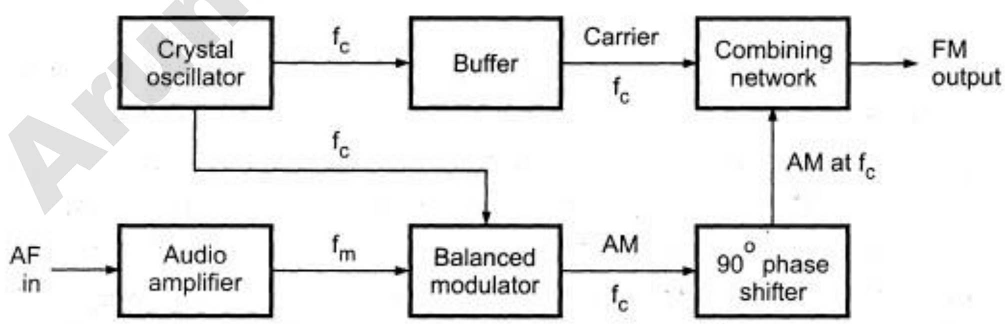
PLL Direct FM transmitter:



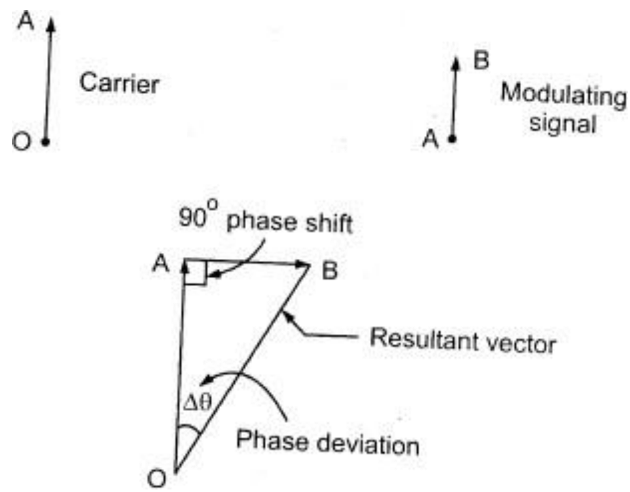
Block diagram of PLL direct FM transmitter

- Fig shows a wide band FM transmitter.
- The VCO o/p freq is divided by N & fed bark to the PLL phase comparator, where it is compared to a stable reference freq.
- The phase comparator generator a correction voltage that is proportional to the difference b/w the 2 frequencies.
- The correction voltage is added to the modulating signal & applied to the VCO i/p.
- The correction voltage adjusts the VCO centre freq to its proper value.
- The LPF prevents the changes in the VCO o/p frequency due to the modulating signal from being converted to a voltage & fed back to VCO.
- The LPF also prevents the loop from locking onto a side frequency.

Indirect Fm transmitter



Block diagram of Armstrong method to generate FM



- Here the modulating signal directly deviates the phase of the carrier, which indirectly changes the frequency.
- The carrier source is a crystal oscillator hence stability can be achieved without a AFC.
- A carrier is phase shifted to 90^0 & fed to the Balanced modulator. Where it is mixed with the i/p modulating signal.
- The o/p of balanced modulator is DSBSC.
- The o/p of Balanced modulator is combined with original carrier in the combining N/W. to produce a low index, phase modulated wavefrom.
- Fig (b) shows phasor of original carrier, modulating signal and the resultant Vector.
- Fig (b) shows the phasors for the side freq. components of the suppressed carrier wave. As suppressed carrier is out of phase with Vc, the upper & Lower side bands combine to produce Vm – 90^o with Vc.
- The phase modulated signal is obtained by vector addition of carrier and modulating signal.
- Modulating signal vector adds to the carrier OA with 90^0 phase Shift.
- The resultant phase modulated vector is OB with phase shift θ.
- This works only if both have the same frequency. The means carrier & modulating signal should have same frequency. Under this condition phase modulation produces FM o/p.

FM Demodulators / Detectors

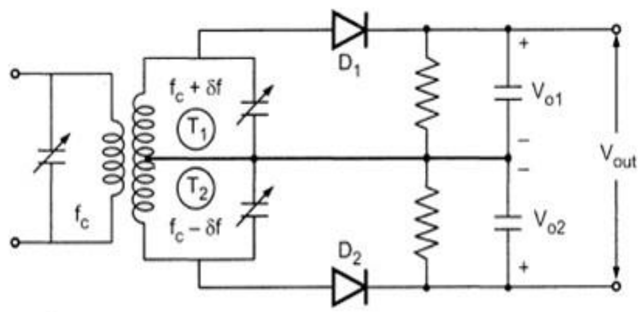
FM demodulator must satisfy the following requirements

- It must convert the frequency variations into amplitude variations
- This conversion must be linear and efficient.
- The demodulator circuits must be insensitive to amplitude changes.
- It should not be too critical in its adjustment and operation.

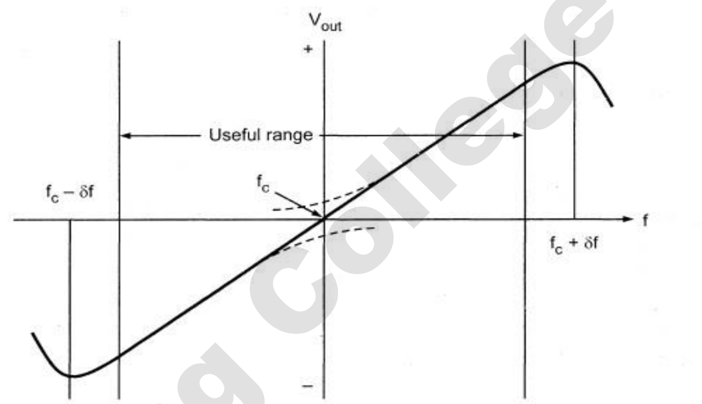
Types of FM demodulator:

- Round Travis Detector or Balanced discriminator.
- Foster – Seley Discriminator or Phase discriminator.
- Ratio Detector.

Slope Detector / Round Travis Detector :



Balanced slope detector



Characteristic of balanced slope detector, or 'S'-curve

- Consists of 2 identical circuit connected back to back.
- FM signal is applied to the tuned LC circuit.
- Two tuned LC circuits are connected in series.
- The inductance of the secondary tuned LC circuit is coupled with the inductance of the primary LC circuit this forms a tuned transformer.
- Upper tuned circuit is T_1 & lower tuned circuit is T_2.
- I/P side LC is tuned to be

  T_1 is tuned to $f_c + \delta f$ - max freq fm.

  T_2 is tuned to $f_c - \delta f$ - max freq fm.

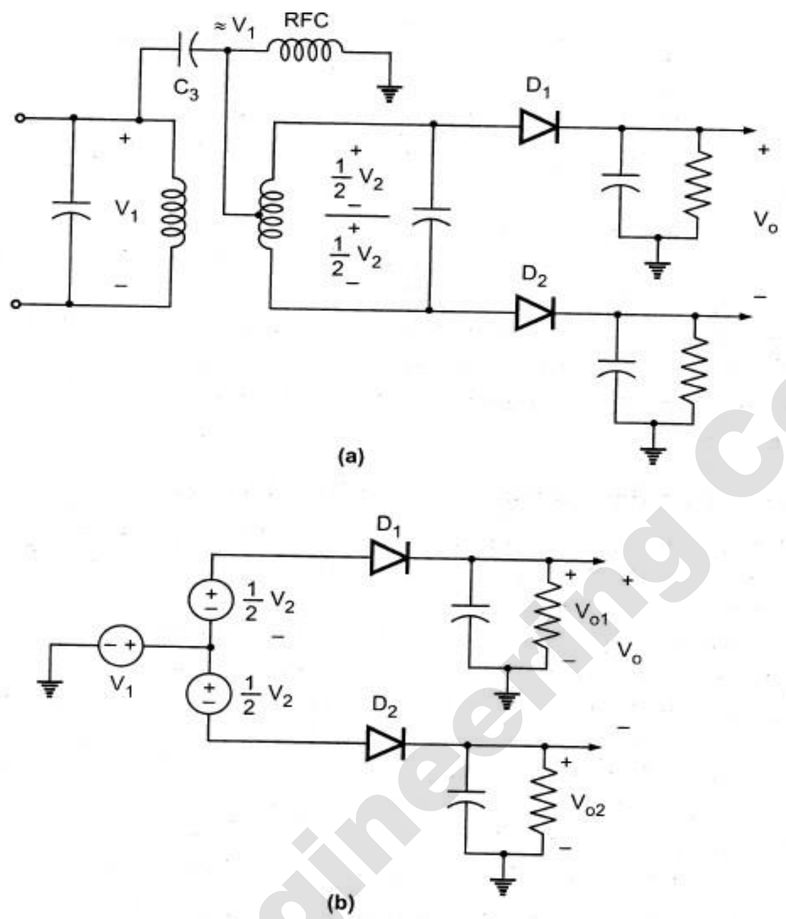
- Secondary of T_1 & T_2 are connected to diodes D_1 & D_2 with RC loads.
- The total o/p is equal to difference b/w Vo1 & Vo2.
- When i/p freq is fc, both T1 & T2 produce the same voltage hence o/p = 0
- When i/p freq is $f_c + \delta f$, the upper circuit T_1 produces maximum voltage since it is tuned to this freq. Hence this produces maximum votalge.

  V_{01} is high compared to V_{02}.

  Vout = $V_{01} - V_{02}$ is positive for fc + δf.

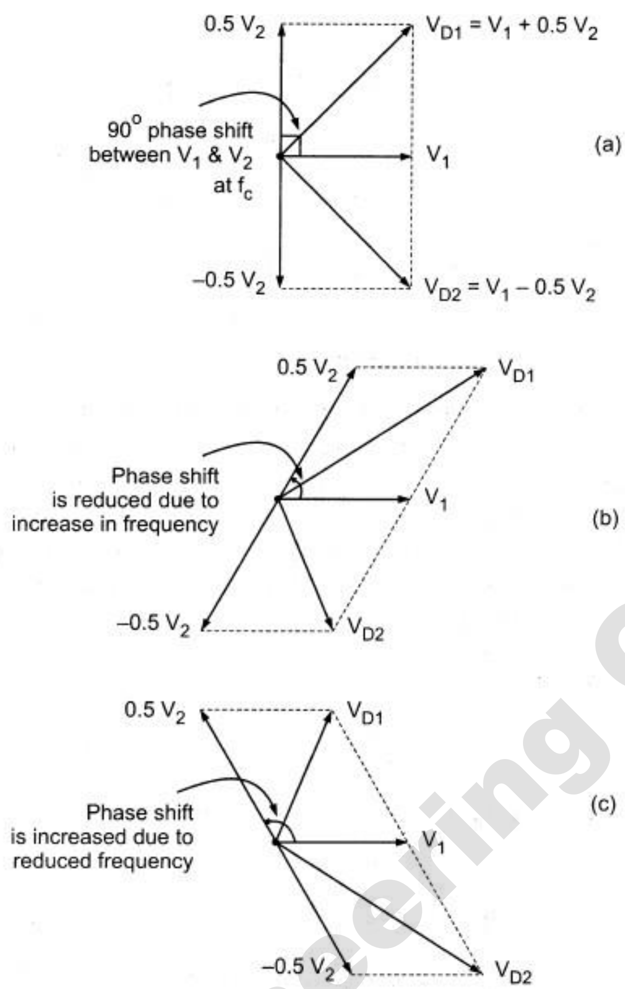
- When i/p freq is $f_c - \delta f$. T_2 produces maximum signal since it is tuned to it. But T_1 produces minimum voltage. Hence o/p Volt = $V_{01} - V_{02}$ is negative. Thus we get a modulating signal.

Foster - Seeley Discriminator :



(a) Basic Foster–Seeley discriminator
(b) Voltage generator equivalent circuit

- The primary voltage is coupled through C_3 & RFC to the centre tap on the secondary.
- The capacitor C3 passes all the frequencies of Fm. The voltage V_1 is generated across RFC.
- RFC offers high impedance to frequencies of Fm.



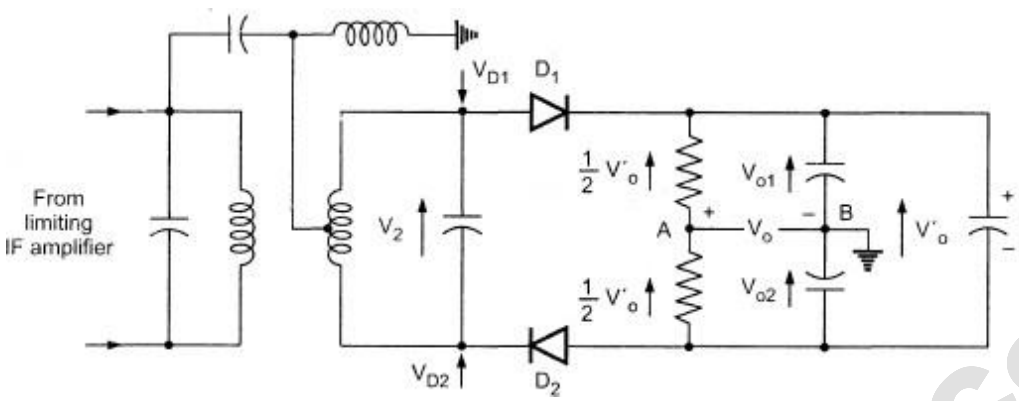
(a) At center frequency, phase shift between V_1 and V_2 is 90°. Hence $|V_{D1}| = |V_{D2}|$

(b) For the frequencies above center frequency, the phase shift between V_1 and V_2 is reduced. Hence $|V_{D1}| > |V_{D2}|$

(c) For frequencies below center frequency, the phase shift between V_1 and V_2 is increased. This makes $|V_{D1}| < |V_{D2}|$

- The voltage V_1 thus appears across centre tap of secondary and ground also.
- The voltage of secondary is V_2 & equally divided across upper half & lower half of the secondary.
- In the figure the voltage across diode D_1 is $V_{DI} = V_1 + 0.5\ V_2$ and that across D_2 is $V_{D2} = V_1 + 0.5\ V_2$.
- The o/p of upper rectifier is V_{01} and lower rectifier is V_{02}.
- The net o/p $V_0 = V_{01} – V_{02} \cong V_0 = |\ V_{D1}\ | - |\ V_{D2}\ |$
- At carrier frequency V_{D1} x V_{D2} are equal hence the net o/p of the discriminator will be zero.
- When the i/p frequency increases above fc the phase shift b/w V_1 & V_2 reduces $|\ V_{D1}\ | > |\ V_{D2}\ |$ hence $V_{01} = |\ V_{D1}\ | - |\ V_{D2}\ |$ will be +ve.
- When the i/p frequency reduces below fc then $|\ V_{D1}\ | > |\ V_{D2}\ |$ hence $V_{01} = |\ V_{D1}\ | - |\ V_{D2}\ |$ will be –ve.

Ratio detector :



Ratio detector can be obtained by sight modifications in the foster-Seeley discriminator. Fig shows the circuit diagram of ratio detector. As shown in the diagram the diode D_2 is reversed, and output is taken from different points. In the above circuit the regular conversion from frequency to phase shift and phase shift to amplitude takes place as in faster–Seeley discriminator. The polarity of voltage in the lower capacitor is reversed. Hence the voltages V_{01} and V_{02} across two capacitors add. (Note that these voltages subtract in faster-seely circuit). And we know that when V_{01} increases, V_{02} decreases and vice-versa as we have seen in faster-Seeley circuit. Since, V_0' is sum of V_{01} and V_{02}, it remains constant. From the circuit of Fig we can write two equations for the output voltage V_0 (Note that V_0 is the net output voltage and taken across points A and B). The First equation will be,

$$V_0 = \frac{1}{2} V'_0 - V_{02}$$

and $V_0 = - \frac{1}{2} V'_0 - V_{01}$

adding the above two equations,

$$2 V_0 = V_{01} - V_{02}$$

$$V_0 = \frac{1}{2} (V_{01} - V_{02})$$

Since $V_{01} \approx | V_{D1}|$ and $| V_{02} |$ above equation will be,

$$V_0 = \frac{1}{2} (| V_{D1} | - | V_{D2} |)$$

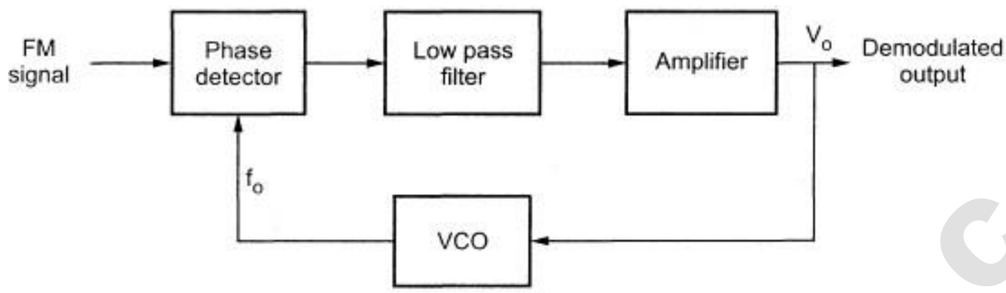
- Here V_{D1} & V_{D2} are obtained as discussed earlier in foster seeley circuit.
- From the equation we know that the output of ratio detector is half compared to that of Foster-Seeley circuit

- As frequency increases above fc' | V_{D1} | > | V_{D2}| hence o/p is +ve.
- III rly as frequency decreases below fc = | V_{D2} | > | V_{D1}|, hence o/p is –ve.

Advantage :

- Reduced fluctuations in the o/p voltage compared to foster seeley circuit

PLL Demodulator circuit



- Fig. shows the block diagram of PLL FM demodulator.
- The output frequency of VCO is equal to the frequency of unmodulated carrier.
- The phase detector generates the voltage which is proportional to difference between the FM signal and VCO output.
- This voltage is filtered and amplified. It is the required modulating voltage.
- Here frequency correction is not required in VCO since it is already done at transmitter

Comparison of FM and AM

	Amplitude Modulation	**Frequency Modulation**
1.	Amplitude of the carrier is varied according to amplitude of modulating signal.	Frequency of the carrier is varied according to amplitude of the modulating signal.
2.	Am has poor fidelity due to narrow bandwidth.	Since the bandwidth is large, fidelity is better.
3.	Most of the power is in carrier hence less efficient.	All the transmitted power is useful.
4.	Noise interference is more.	Noise interference is minimum.
5.	Adjacent Chennai Interference is present	Adjacent Chennai Interference is avoided due to guard bands.

6.	Am Broadcast operates in MF and HF range	FM Broadcost operates in VHF and UHF range.
7.	In Am only carrier and two sidebands are present	Infinite number of sidebands are present
8.	The transmission equipment is simple.	The transmission equipment is complex.
9.	Transmitted power varies according to modulation index.	Transmitted power remains constant irrespective of modulation index.
10.	Depth of modulation have limitation. It can not be increased above I.	Depth of modulation have no limitation. It can be increased by increasing frequency deviation.

UNIT 3 & 4

1. Determine the irreducible limit below which a signal cannot be compressed.

2. Deduce the ultimate transmission rate for reliable communication over a noisy channel.

3. Define Channel Capacity - the intrinsic ability of a channel to convey information.

Communication system

The function of any communication system is to convey the information from source to destination.

Discrete message

Message which is selected from a finite number of predetermined messages.

During one time one message is transmitted. During the next time interval the next from the set is transmitted.

Memory source

A source with memory for which each symbol depends on the previous symbols.

Memoryless source

Memoryless in the sense that the symbol emitted at any time is independent of previous choices.

Probabilistic experiment involves the observation of the output emitted by a discrete source during every unit of time. The source output is modeled as a discrete random variable, S,which takes on symbols form a fixed finite alphabet.

$$S = s0, s1, s2, \cdots , sk\text{-}1$$

with probabilities

$$P(S = sk) = pk,\ k = 0, 1, \cdots ,K - 1$$

We assume that the symbols emitted by the source during successive signaling intervals are statistically independent. A source having the properties is described is called discrete memoryless source, memoryless in the sense that the symbol emitted at any time is independent of previous choices.

Source alphabet

The set of source symbols is called the source alphabet.

Symbols or letters

The element of the set is called symbol.

Uncertainty

The amount of information contained in each symbols is closely related to its uncertainty or surprise.

If an event is certain (that is no surprise, of probability 1) it conveys zero information.

We can define the amount of information contained in each symbols.

$$I(s_k) = \log_b(1/\ p_k)$$

Here, generally use log2 since in digital communications we will be talking about bits. The above expression also tells us that when there is more uncertainty(less probability) of the symbol being occurred then it conveys more information.

Unit of the information

The unit of the information depends on the base of the logarithmic function.

UNIT	b VALUE
Binit	2
Decit (OR) Hartley	10
Natural unit(nat)	e

When pk = ½ ,we have I(sk) = 1 bit.

Some properties of information are summarized here:

1. For certain event i.e, pk = 1 the information it conveys is zero, I(sk) = 0. Absolutely certain of the outcome of an event.
2. For the events 0 ≤ pk ≤ 1 the information is always I(sk) ≥0.Either provides some or no information, but never brings about a loss of information.
3. If for two events pk > pi, the information content is always I(sk) < I(si). The less probable the event is, the more information we gain when it occurs.

4. I(sksi) = I(sk)+I(si) if sk and si are statistically independent.

   Proof:

   $p(s_j, s_k) = p(s_j)\, p(s_k)$ if sk and si are statistically independent.

   $I(s_j, s_k) = \log(1 / p(s_j, s_k))$

   $= \log(1 / p(s_j)\, p(s_k))$

   $= \log(1 / p(s_j)) + = \log(1 / p(s_k))$

   $= I(s_j)\, I(s_k)$

Average information or entropy

The amount of information I(sk) produced by the source during an arbitrary signalling interval depends on the symbol sk emitted by the source at that time. Indeed, I(sk) is a discrete random variable that takes on the values I(s0), I(s1), · · · , I(sK-1) with probabilities p0, p1, · · · , pK-1 respectively.

The mean of I(sk) over the source alphabet S is given by

$$
\begin{aligned}
H(\mathcal{S}) &= E[I(s_k)] \\
&= \sum_{k=0}^{K-1} p_k I(s_k) \\
&= \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right)
\end{aligned}
$$

The important quantity H(φ) is called the entropy of a discrete memoryless source with source alphabet φ. It is a measure of the average information content per source symbol. Note the entropy H(φ) depends only on the probabilities of the symbols in the alphabet φ of the source.

Problem

1. A DMS has four symbols S_1, S_2, S_3 S_4 with probabilities 0.40, 0.30, 0.20, 0.10
   a. Calculate H(φ).
   b. Find the amount of information contained in the message $S_1S_2S_3\,S_4$ and $S_4S_3S_3\,S_2$, and compare with the H(φ).

Solution

a. $H(\varphi) = \sum Pk \log_2(1/ Pk)$

   $= -\,0.4 \log_2 0.4 - 0.3 \log_2 0.3 - 0.2\log_2 0.2 - 0.1\log_2 0.1$

   $= 1.85$ b/symbol

b. $P(S_1S_2S_3\,S_4) = (0.4)(0.3)(0.2)(0.1) = 0.0096$

$I(S_1S_2S_3\,S4) = -\log_2(0.0096) = 0.60$ b/symbol

$I(S_1S_2S_3\,S4) < 7.4\ (4\,H(\varphi))$

$P(S_4S_3S_3\,S_2) = (0.1)(0.2)^2\,(0.3) = 0.0012$

$I(S_4S_3S_3\,S_2) = -\log_2(0.0012) = 9.70$ b/ symbol

$I(S_4S_3S_3\,S_2) > 7.4\ (4\,H(\varphi))$

Some properties of entropy

The entropy $H(\varphi)$ of a discrete memoryless source is bounded as follows:

$$0 \le H(\varphi) \le \log 2(K)$$

where K is the radix (number of symbols)of the alphabet S of the source.

Furthermore, we may make two statements:

1. $H(\varphi) = 0$, if and only if the probability pk = 1 for some k, and the remaining probabilities in the set are all zero; this lower bound on entropy corresponds to no uncertainty.

2. $H(\varphi) = \log 2(K)$, if and only if pk = 1/ K for all k; this upper bound on entropy corresponds to maximum uncertainty.

Proof:

$$H(\varphi) \ge 0.$$

Since each probability pk is less than or equal to unity, it follows that each term

Pk $\log_b(1/\,p_k)$ is always nonnegative. So $H(\varphi) \ge 0$.

The term Pk $\log_b(1/\,p_k)$ is zero if, and only if, pk = 0 or 1. That is pk =1 for some k and all the rest are zero.

$$H(\varphi) \le \log 2(K)$$

To prove this upper bound , we make use of a property of the natural logarithm.

$$\log x \le x - 1, \qquad x \ge 0$$

To proceed with this proof, consider any two probability distributions {p0, p1, · · · , pk-1 } and {q0, q1, q2, · · · , qk-1 } on the alphabet φ = {s0, s1, s2, · · · , sk-1 } of a discrete memoryless source. Then changing to the natural logarithm, we may write

$$\sum_{k=0}^{K-1} p_k \log_2\left(\frac{q_k}{p_k}\right) = \frac{1}{\log 2} \sum_{k=0}^{K-1} p_k \log\left(\frac{q_k}{p_k}\right)$$

Hence, using the inequality, we get

$$\begin{aligned}\sum_{k=0}^{K-1} p_k \log_2\left(\frac{q_k}{p_k}\right) &\leq \frac{1}{\log 2} \sum_{k=0}^{K-1} p_k\left(\frac{q_k}{p_k} - 1\right) \\ &\leq \frac{1}{\log 2} \sum_{k=0}^{K-1} (q_k - p_k) \\ &\leq \frac{1}{\log 2}\left(\sum_{k=0}^{K-1} q_k - \sum_{k=0}^{K-1} p_k\right) = 0\end{aligned}$$

We thus have the fundamental inequality

$$\sum_{k=0}^{K-1} p_k \log_2\left(\frac{q_k}{p_k}\right) \leq 0$$

Where the equality holds only if pk = qk for all k. Suppose we next put

$$q_k = \frac{1}{K}, \qquad k = 0, 1, \ldots, K-1$$

$$\sum_{k=0}^{K-1} q_k \log_2\left(\frac{1}{q_k}\right) = \log_2 K$$

$$\sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right) \leq \log_2 K$$

So, $$H(\mathcal{S}) \leq \log_2 K$$

Thus H(φ) is always less than or equal to $\log_2$k. This equality holds only if the symbols are equiprobable.

Entropy of a binary memoryless channel

Consider a discrete memoryless binary source shown defined on the alphabet φ = {0, 1}. Let the probabilities of symbols 0 and 1 be p0 and 1- p0 respectively.

The entropy of this channel is given by

$$H(\mathcal{S}) = -p_0 \log_2 p_0 - p_1 \log_2 p_1$$
$$= -p_0 \log_2 p_0 - (1 - p_0) \log_2(1 - p_0) \text{ bits}$$

From which we observe the following:

1. When $p_0=0$, the entropy $H(\varphi)=0$.
2. When $p_0=1$, the entropy $H(\varphi)=1$.
3. The entropy attains its maximum value, $H_{max} = 1$ bit, $p_0= p_1= 1/2$., that is, symbols 1 and 0 are equally probable.

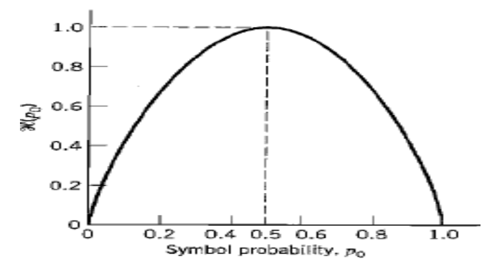


Figure - Entropy of a discrete memoryless binary source

Information rate

If the source of the message generates messages at the rate of r messages per second, then the information rate is defined to be

R= rH = average number of bits of information / second.

Example problem

An analog signal is bandlimited to B Hz,sampled at the nyquist rate, and the samples are quantized into four levels. The quantization levels Q1,Q2,Q3,Q4(messages) are assumed independent and occur with probabilities P1=P4=1/8 and P2=P3=3/8. Find the information rate of the source.

Solution

The average information H is

$$H= p_1\log_2(1/p_1) + p_2\log_2(1/p_2) + p_3\log_2(p_3) + p_4\log_2(1/ p_4)$$

$$=1/8 \log_2(8) +3/8 \log_2(8/3) +3/8 \log_2(8/3) +1/8 \log_2(8)$$

= 1.8 bits / message

The information rate R is

R= rH =2B(1.8) = 3.6 bits /s.

Shannon source coding theorem

An important problem in communication is the efficient representation of data generated by a discrete source. The process by which this representation is accomplished is called source encoding. The device that performs that representation is called a source encoder.

Variable length code

If some source symbols are known to be more probable than others, then the source code is generated by assigning short code to frequent source symbols, and long code to rare source symbols.

EX : Morse code, in which the letters and alphabets are encoded into streams of marks and spaces, denoted as dots "." And dashes "-".

Our primary interest is in the development of an efficient source encoder that satisfies two functional requirements:

1. The code words produced by the encoder are in binary form.

2. The source code is uniquely decodable, so that the original source sequence can be reconstructed perfectly from the encoded binary sequence.

We define the average code word length, L, of the source encoder as

$$L = \sum_{k=0}^{K-1} p_k I_k$$

In physical terms, the parameter L represents the average number of bits per source symbol used in the source encoding process. Let L_{min} denote the minimum possible value of L. We then define the coding efficiency of the source encoder as

$$\eta = L_{min} / L$$

The source encoder is said to be efficient when η approaches unity.

Source coding theorem:

Given a discrete memoryless source of entropy $H(\varphi)$, the average code-word length L for any distortionless source encoding scheme is bounded as

$$L \geq H(\varphi)$$

According to the source-coding theorem, the entropy $H(\varphi)$ represents a fundamental limit on the average number of bits per source symbol necessary to represent a discrete memoryless source in that it can be made as small as, but no smaller than, the entropy $H(\varphi)$. Thus with Lmin = $H(\varphi)$, we may rewrite the efficiency of a source encoder in terms of the entropy $H(\varphi)$ as

$$\eta = H(\varphi)/ L$$

Data Compaction:

1. Removal of redundant information prior to transmission.

2. Lossless data compaction – no information is lost.

3. A source code which represents the output of a discrete memoryless source should be uniquely decodable.

Prefix Coding

Consider a discrete memoryless source of alphabet $\{s_0, s_1, s_2, \cdots, s_{k-1}\}$

and statistics $\{p_0, p_1, p_2, \cdots, p_{k-1}\}$.

For each finite sequence of symbols emitted by the source, the corresponding sequence of code words is different from the sequence of code words corresponding to any other source sequence. For the above mentioned symbol, let the code word be denoted by

$$\{m_{k0}, m_{k1}, m_{k2}, \cdots, m_{kn-1}\}$$

– the element are 0s and 1s.

n - denotes the code word length

Prefix condition

The initial part of the code word is represented by the elements $\{m_{k0}, m_{k1}, m_{k2}, \cdots, m_{ki}\}$
Any sequence made up of the initial part of the code word is called prefix.

Prefix code

1. The Prefix Code is variable length source coding scheme where no code is the prefix of any other code.

2. The prefix code is a uniquely decodable code.

3. But, the converse is not true i.e., all uniquely decodable codes may not be prefix codes.

Table 1: Illustrating the definition of prefix code

Symbol	Prob.of Occurrence	Code I	Code II	Code III
s_0	0.5	0	0	0
s_1	0.25	1	10	01
s_2	0.125	00	110	011
s_3	0.125	11	111	0111

From 1 we see that Code I is not a prefix code. Code II is a prefix code. Code III is also uniquely decodable but not a prefix code. Prefix codes also satisfies Kraft-McMillan inequality which is given by

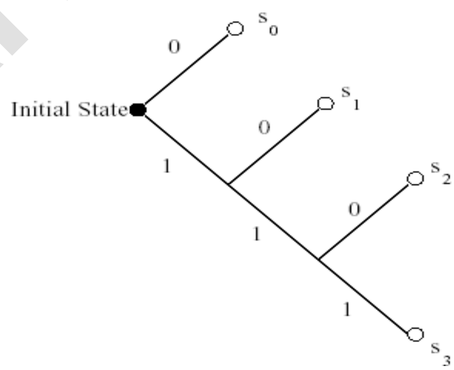
$$\sum_{k=0}^{K-1} 2^{-l_k} \leq 1$$

Code I violates the Kraft – McMillan inequality.

Both codes II and III satisfies the Kraft – McMillan inequality, but only code II is a prefix code.

Decoding procedure

1. The source decoder simply starts at the beginning of the sequence and decodes one codeword at the time.
2. The decoder always starts at the initial state of the tree.
3. The received bit moves the decoder to the terminal state if it is 0,or else to next decision point if it is 1.



Given a discrete memoryless source of entropy H(φ), a prefix code can be constructed with an average code-word length l, which is bounded as follows:

$$H(\varphi) \leq L \leq H(\varphi)+1$$

The left hand side of the above equation, the equality is satisfied owing to the condition that, any symbol sk is emitted with the probability

$$p_k = 2^{-l_k}$$

where, lk is the length of the codeword assigned to the symbol sk.

Shannon – fano coding

Procedure

1. List the symbols in order of decreasing probability.
2. Partition the set into two sets that are as close to equiprobable as possible, and assign 0 to the upper set and 1 to the lower set.
3. Continue the process, each time partitioning the sets with nearly equal probabilities as possible until further partitioning not possible.

Example problems

1. A DMS has six symbols S_1 , S_2, S_3 S_4, S_5, S_6, with corresponding probabilities 0.30, 0.25, 0.20, 0.12, 0.08, 0.05. construct a Shannon – fano code for S.

Sk	pk	Step 1	Step 2	Step 3	Step 4	Step 5
S1	0.30	0	0			00
S2	0.25	0	1			01
S3	0.20	1	0			10
S4	0.12	1	1	0		110
S5	0.08	1	1	1	0	1110
S6	0.05	1	1	1	1	1111

2. A DMS has six symbols S_1 , S_2, S_3, S_4 with corresponding probabilities, 1/2 , 1/4, 1/8, 1/8, construct a Shannon – fano code for S.

Sk	pk	Step 1	Step 2	Step 3	Step 4
S1	1/2	0			0
S2	1/4	1	0		10
S3	1/8	1	1	0	110
S4	1/8	1	1	1	111

3. A DMS has five equally likely symbols S_1 , S_2, S_3 S_4, S_5 construct a Shannon – fano code for S.

Sk	pk	Step 1	Step 2	Step 3	Step 4
S1	0.20	0	0		00
S2	0.20	0	1		01
S3	0.20	1	0		10
S4	0.20	1	1	0	110
S5	0.20	1	1	1	111

Huffman coding

It is to assign to each symbol of an alphabet a sequence of bits roughly equal in length to the amount of information conveyed by the symbol in question.

Algorithm

1. The source symbols are listed in order of decreasing probability. The two source symbols of lowest probability are assigned a 0 and a 1. This part of the step is reffered to as a splitting stage.

2. These two source symbols are regarded as being combined into a new source symbol with probability equal to the sum of the original probabilities. The probability of the new symbol is placed in the list in accordance with its value.

3. The procedure is repeated until we are left with a final list of source statistics of only two for which a 0 and a 1 are assigned.

Symbol	Stage I	Stage II	Stage III	Stage IV
S0,	0.4	0.4	0.4	0.6 0
S1	0.2	0.2	0.4 0	0.4 1
S2	0.2	0.2 0	0.2 1	
S3	0.1 0	0.2 1		
S4	0.1 1			

The code for each source symbol is found by working backward and tracing the sequence of 0s and 1s assigned to that symbol as well as its successors.

Symbol	probability	code word
S0,	0.4	00
S1	0.2	10
S2	0.2	11
S3	0.1	010
S4	0.1	011

Drawbacks:

1. Requires proper statistics.

2. Cannot exploit relationships between words, phrases etc.,

3. Does not consider redundancy of the

language. **Lempel-ziv coding**

1. Overcomes the drawbacks of Huffman coding

2. It is an adaptive and simple encoding scheme.

3. When applied to English text it achieves 55% in contrast to Huffman coding which achieves only 43%.

4. Encodes patterns in the text This algorithm is accomplished by parsing the source data stream

into segments that are the shortest subsequences not encountered previously.

Problems

Let the input sequence be

000101110010100101. ....

We assume that 0 and 1 are known and stored in

codebook

subsequences stored : 0, 1

Data to be parsed: 000101110010100101. ....

is 00

subsequences stored: 0, 1, 00

Data to be parsed: 0101110010100101. ....

The second shortest subsequence not seen before is 01; accordingly, we go on to

write Subsequences stored: 0, 1, 00, 01

Data to be parsed: 01110010100101. ....

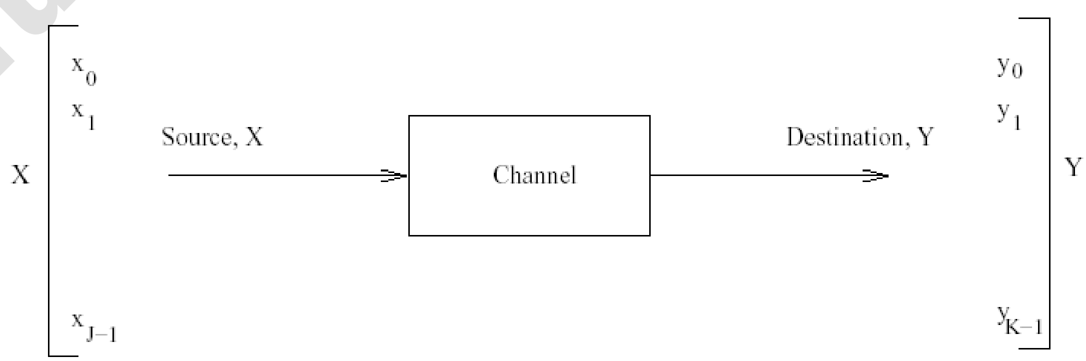
We continue in the manner described here until the given data stream has been completely parsed. The

code book is shown below:

Numerical positions:	1	2	3	4	5	6	7	8	9
subsequences:	0	1	00	01	011	10	010	100	101
Numerical Repre sentations:			11	12	42	21	41	61	62
Binary encoded blocks:			0010	0011	1001	0100	1000	1100	1101

Discrete Memoryless Channels

Let X and Y be the random variables of symbols at the source and destination respectively. The description of the channel is shown in the Figure



The channel is described by an input alphabet

$$\{x_0, x_1, x_2, \cdots, x_{J-1}\}, \text{ J – input alphabet size}$$

an output alphabet

$$\{y_0, y_1, y_2, \cdots, y_{K-1}\}, \text{ K – output alphabet size}$$

and a set of transition probabilities

$$p(y_k/x_j) = p(Y= y_k / X= x_j) \text{ for all j and k}$$

channel matrix or transition matrix

$$P = \begin{bmatrix} p(y_0/x_0) & p(y_1/x_0) & \cdots\cdots\; p(y_{K-1}/x_0) \\ p(y_0/x_1) & p(y_1/x_1) & \cdots\cdots\; p(y_{K-1}/x_1) \\ . & & \\ . & & \\ . & & \\ p(y_0/x_{J-1}) & p(y_1/x_{J-1}) & \cdots\cdots\; p(y_{K-1}/x_{J-1}) \end{bmatrix}$$

each row corresponds to a fixed channel input.

each column corresponds to a fixed channel output.

Input probability distribution $p(x_j)$, j=1,2,……J-1, the event that the channel input $X= x_j$

occurs with probability

$$p(x_j) = p(X=x_j) \text{ for all j.}$$

joint probability distribution

$$p(x_j, y_k) = p(X=x_j, Y= y_k)$$

$= p(y_k/x_j)\ p(x_j)$

Marginal probability distribution of the output random variable Y is obtained by averaging out the dependence of $p(x_j, y_k)$ on x_j,

$$\begin{aligned} p(y_k) &= P(Y = y_k) \\ &= \sum_{j=0}^{J-1} P(Y = y_k | X = x_j) P(X = x_j) \\ &= \sum_{j=0}^{J-1} p(y_k | x_j) p(x_j) \qquad \text{for } k = 0, 1, \ldots, K-1 \end{aligned}$$

Binary Symmetric Channel

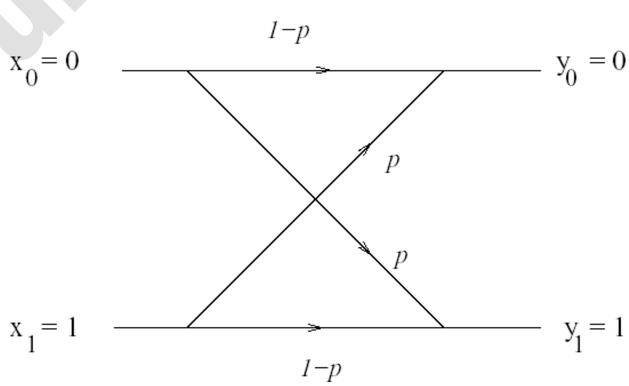
– A discrete memoryless channel with J = K = 2.

– The Channel has two input symbols($x_0 = 0$, $x_1 = 1$)and two output symbols($y_0 = 0$, $y_1 = 1$).

– The channel is symmetric because the probability of receiving a 1 if a 0 is sent is same as the probability of receiving a 0 if a 1 is sent.

– The conditional probability of error is denoted by p. Abinary symmetric channel is shown in Figure and its transition probability matrix is given by

$$\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}$$



Mutual Information

If the output Y as the noisy version of the channel input X and H(X) is the uncertainity associated with X, then the uncertainity about X after observing Y , H(X|Y) is given by

$$\begin{aligned} H(\mathcal{X}|\mathcal{Y}) &= \sum_{k=0}^{K-1} H(\mathcal{X}|Y=y_k)p(y_k) && (1) \\ &= \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j|y_k)p(y_k)log_2\left[\frac{1}{p(x_j|y_k)}\right] && (2) \\ &= \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j,y_k)log_2\left[\frac{1}{p(x_j|y_k)}\right] && (3) \end{aligned}$$

The quantity H(*X*|Y) is called Conditional Entropy. It is the amount of uncertainity about the channel input after the channel output is observed. Since H(X) is the uncertainity in channel input before observing the output, H(X) - H(X|Y) represents the uncertainity in channel input that is resolved by observing the channel output. This uncertainity measure is termed as Mutual Information of the channel and is denoted by I(X; Y).

$$I(\mathcal{X};\mathcal{Y}) = H(\mathcal{X}) - H(\mathcal{X}|\mathcal{Y}) \qquad (4)$$

$$I(\mathcal{Y};\mathcal{X}) = H(\mathcal{Y}) - H(\mathcal{Y}|\mathcal{X}) \qquad (5)$$

Where the H(Y) is the entropy of the channel output and H(Y/X) is the conditional entropy of the channel output given the channel input.

Properties of Mutual Information

Property 1:

The mutual information of a channel is symmetric, that is

$$I(\mathcal{X};\mathcal{Y}) = I(\mathcal{Y};\mathcal{X}) \qquad (6)$$

I(X; Y) = I(Y;X)

Where the mutual information I(X; Y) is a measure of the uncertainty about the channel input that is resolved by observing the channel output, and the mutual information I(Y;X) is a measure of the uncertainty about the channel output that is resolved by sending the channel output.

Proof:

$$\begin{aligned}
H(\mathcal{X}) &= \sum_{j=0}^{J-1} p(x_j) \log_2 \left[\frac{1}{p(x_j)}\right] && (7)\\
&= \sum_{j=0}^{J-1} p(x_j) \log_2 \left[\frac{1}{p(x_j)}\right] \sum_{k=0}^{K-1} p(y_k|x_j) && (8)\\
&= \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(y_k|x_j) p(x_j) \log_2 \left[\frac{1}{p(x_j)}\right] && (9)\\
&= \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j, y_k) \log_2 \left[\frac{1}{p(x_j)}\right] && (10)
\end{aligned}$$

Substituting Eq.3 and Eq.10 in Eq.4 and then combining, we obtain

$$I(\mathcal{X};\mathcal{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j, y_k) \log_2 \left[\frac{p(x_j|y_k)}{p(x_j)}\right] \qquad (11)$$

From Bayes' rule for conditional probabilities, we have

$$\frac{p(x_j|y_k)}{p(x_j)} = \frac{p(y_k|x_j)}{p(y_k)} \qquad (12)$$

Hence, from Eq.11 and Eq.12

$$I(\mathcal{X};\mathcal{Y}) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} p(x_j, y_k) \log_2 \left[\frac{p(y_k|x_j)}{p(y_k)}\right] = I(\mathcal{Y};\mathcal{X}) \quad (13)$$

Property 2:

The mutual is always non-negative, that is I(X; Y) ≥0

Proof:

We know,

$$p(x_j|y_k) = \frac{p(x_j, y_k)}{p(y_k)} \qquad (14)$$

Substituting Eq. 14 in Eq. 13, we get

$$I(\mathcal{X};\mathcal{Y}) = \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j, y_k)\log_2\left[\frac{p(x_j, y_k)}{p(x_j)p(y_k)}\right] = I(\mathcal{Y};\mathcal{X}) \qquad (15)$$

Using the following fundamental inequality which we derived discussing the properties of Entropy,

$$\sum_{k=0}^{K-1} p_k \log_2\left(\frac{q_k}{p_k}\right) \leq 0$$

Drawing the similarities between the right hand side of the above inequality and the left hand side of Eq. 13, we can conclude that

$$I(\mathcal{X};\mathcal{Y}) \geq 0$$

With equality if, only if,

$p(x_j|y_k) = p(x_j)p(y_k)$ for all j and k.

Property 2 states that we cannot lose information, on the average, by observing the output of a channel. Moreover, the mutual information is zero if, and only if, the input and output symbols of the channel are statistically independent.

Property 3:

The mutual information of a channel is related to the joint entropy of the channel input and channel output by

$$I(\mathcal{X};\mathcal{Y}) = H(\mathcal{X}) + H(\mathcal{Y}) - H(\mathcal{X},\mathcal{Y})$$

where, the joint entropy (X, Y) is defined as

$$H(\mathcal{X},\mathcal{Y}) = \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left(\frac{1}{p(x_j,y_k)}\right)$$

Proof:

$$H(\mathcal{X},\mathcal{Y}) = \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left(\frac{p(x_j)p(y_k)}{p(x_j,y_k)}\right) + \qquad (16)$$

$$\sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left(\frac{1}{p(x_j)p(y_k)}\right) \qquad (17)$$

$$= I(\mathcal{X};\mathcal{Y}) + \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left(\frac{1}{p(x_j)p(y_k)}\right) \qquad (18)$$

$$\sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left(\frac{1}{p(x_j)p(y_k)}\right) \qquad (19)$$

$$= \sum_{j=0}^{J-1}\log_2\left(\frac{1}{p(x_j)}\right)\sum_{k=0}^{K-1} p(x_j,y_k) + \qquad (20)$$

$$\sum_{k=0}^{K-1}\log_2\left(\frac{1}{p(y_k)}\right)\sum_{j=0}^{J-1} p(x_j,y_k) \qquad (21)$$

$$= \sum_{j=0}^{J-1} p(x_j)\log_2\left(\frac{1}{p(x_j)}\right) + \sum_{k=0}^{K-1} p(y_k)\log_2\left(\frac{1}{p(y_k)}\right) \qquad (22)$$

$$= H(\mathcal{X}) + H(\mathcal{Y}) \qquad (23)$$

Therefore, from Eq. 18 and Eq. 23, we have

$$H(\mathcal{X},\mathcal{Y}) = -I(\mathcal{X};\mathcal{Y}) + H(\mathcal{X}) + H(\mathcal{Y})$$

Channel Capacity

Channel Capacity, C is defined as ‘the maximum mutual information I(X; Y) in any single use of the channel(i.e., signaling interval), where the maximization is over all possible input probability distributions {$p(x_j)$} on X”

$$C = \max_{p(x_j)} I(\mathcal{X}; \mathcal{Y}) \qquad (1)$$

C is measured in bits/channel-use, or bits/transmission.

Example:

For, the binary symmetric channel discussed previously, I(X; Y) will be maximum when $p(x_0) = p(x_1) = 1/2$. So, we have

$$C = I(\mathcal{X}; \mathcal{Y})\big|_{p(x_0)=p(x_1)=\frac{1}{2}} \qquad (2)$$

Since, we know

$$p(y_0|x_1) = p(y_1|x_0) = p \qquad (3)$$

$$p(y_0|x_0) = p(y_1|x_1) = 1 - p \qquad (4)$$

Using the probability values in Eq. 3 and Eq. 4 in evaluating Eq. 2, we get

$$\begin{aligned} C &= 1 + p \log_2 p + (1-p) \log_2 (1-p) \\ &\implies 1 - H(p) \end{aligned} \qquad (5)$$

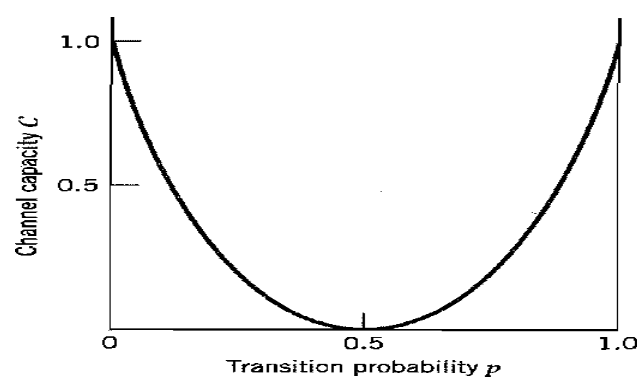


FIGURE-Variation of channel capacity of a binary symmetric channel with transition probability p.

1. When the channel is noise free, p=0, the channel capacity C attains its maximum value of one bit per channel use. At this value the entropy function attains its minimum value of zero.
2. When the conditional probability p=1/2 due to noise, the channel capacity C attains its minimum value of zero,whereas the entropy function attains its maximum value of unity, in such a case the channel is said to be useless.

Channel Coding Theorem:

Goal: Design of channel coding to increase resistance of a digital communication system to channel noise.

Channel coding

Mapping of the incoming data sequence into channel input sequence. It is performed in the transmitter by a channel encoder.

Channel decoding (inverse mapping)

Mapping of the channel output sequence into an output data sequence. It is performed in the receiver by a channel decoder.

The channel coding theorem is defined as process to introduce redundancy in order to reconstruct the original source sequence as accurately as possible.

1. Let a discrete memoryless source

– with an alphabet φ

– with an entropy H(φ)

– produce symbols once every Ts seconds

2. Let a discrete memoryless channel

– have capacity C

– be used once every Tc seconds.

3. Then if,

$$\frac{H(\mathcal{S})}{T_s} \le \frac{C}{T_c}$$

There exists a coding scheme for which the source output can be transmitted over the channel and be reconstructed with an arbitrarily small probability of error. The parameter C / Tc

is called critical rate.

4. Conversly, if

$$\frac{H(\mathcal{S})}{T_s} > \frac{C}{T_c}$$

it is not possible to transmit information over the channel and reconstruct it with an arbitrarily small probability of error.

Example:

Considering the case of a binary symmetric channel, the source entropy H(Φ) is 1. Hence, from the above equation, we have

$$\frac{1}{T_s} \leq \frac{C}{T_c}$$

But the ratio Tc / Ts equals the code rate, r of the channel encoder.

$$r = \frac{T_c}{T_s}$$

$$r \leq C$$

Hence, for a binary symmetric channel, if r ≤ C, then there exists a code capable of achieving an arbitrarily low probability of error.

Information Capacity Theorem:

The Information Capacity Theorem is defined as ‘The information capacity of a continuous channel of bandwidth B hertz, perturbed by additive white Gaussian noise of power spectral Density N0/ 2 and limited in bandwidth to B, is given by

$$C = B \log_2 \left(1 + \frac{P}{N_0 B}\right)$$

where P is the average transmitted power. Proof:

Assumptions:

1. band-limited, power-limited Gaussian channels.
2. A zero-mean stationary process X(t) that is band-limited to B hertz, sampled at Nyquist rate of 2B samples per second
3. These samples are transmitted in T seconds over a noisy channel, also band-limited to B hertz.

The number of samples, K is given by

$$K = 2BT$$

We refer to X_k as a sample of the transmitted signal. The channel output is mixed with additive white Gaussian noise(AWGN) of zero mean and power spectral density N0/2. The noise is band-limited to B hertz. Let the continuous random variables Y_k, k = 1, 2, · · · ,K denote samples of the received signal, as shown by

$$Y_k = X_k + N_k \text{ ,}$$

The noise sample Nk is Gaussian with zero mean and variance given by

$$\sigma^2 = N_0 B$$

The transmitter power is limited; it is therefore

$$E[X_k^2] = P$$

Now, let $I(X_k; Y_k)$ denote the mutual information between X_k and Y_k. The capacity of the channel is given by

$$C = \max_{f_{X_k}(x)} I(X_k; Y_k) : E[X_k^2] = P$$

The mutual information I(Xk; Yk) can be expressed as

$$I(X_k; Y_k) = h(Y_k) - h(Y_k | X_k)$$

This takes the form

$$I(X_k; Y_k) = h(Y_k) - h(N_k)$$

When a symbol is transmitted from the source, noise is added to it. So, the total power is $P + \sigma^2$.

For the evaluation of the information capacity C, we proceed in three stages:

1. The variance of sample Yk of the received signal equals $P + \sigma^2$. Hence, the differential entropy of Y_k is

$$h(Y_k) = \frac{1}{2} log_2[2\pi e(P + \sigma^2)]$$

2. The variance of the noise sample Nk equals σ^2. Hence, the differential entropy of N_k is given by

$$h(N_k) = \frac{1}{2} log_2(2\pi e \sigma^2)$$

3. Now, substituting above two equations into

$$I(X_k; Y_k) = h(Y_k) - h(N_k) \text{ yeilds}$$

$$C = \frac{1}{2} log_2(1 + \frac{P}{\sigma^2})$$ bits per transmission.

The number K equals 2BT. Accordingly, we may express the information capacity in the equivalent form:

$$C = B\log_2(1 + \frac{P}{N_0 B})\, bits\ per\ second$$

UNIT-5

Spread Spectrum and Multiple Access Techniques

Spread Spectrum

- Analog or digital data
- Analog signal
- Spread data over wide bandwidth
- Makes jamming and interception harder
- Frequency hoping
  - Signal broadcast over seemingly random series of frequencies
- Direct Sequence
  - Each bit is represented by multiple bits in transmitted signal
  - Chipping code

Spread Spectrum Concept

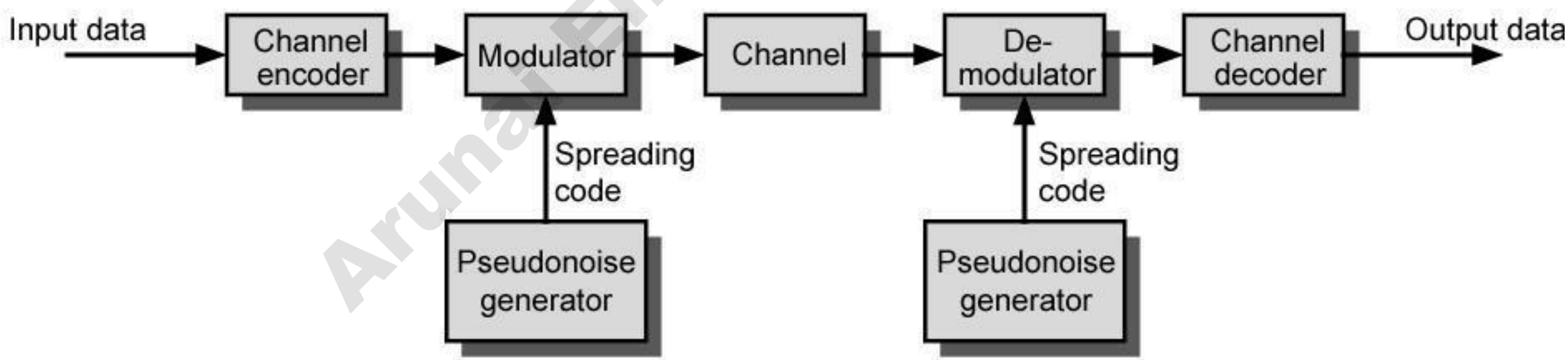
- Input fed into channel encoder
  - Produces narrow bandwidth analog signal around central frequency
- Signal modulated using sequence of digits

Spreading code/sequence

– Typically generated by pseudonoise/pseudorandom number generator

- Increases bandwidth significantly
  - Spreads spectrum
- Receiver uses same sequence to demodulate signal
- Demodulated signal fed into channel decoder

General Model of Spread Spectrum System



Pseudorandom Numbers

- Generated by algorithm using initial seed
- Deterministic algorithm
  - Not actually random
  - If algorithm good, results pass reasonable tests of randomness
- Need to know algorithm and seed to predict sequence

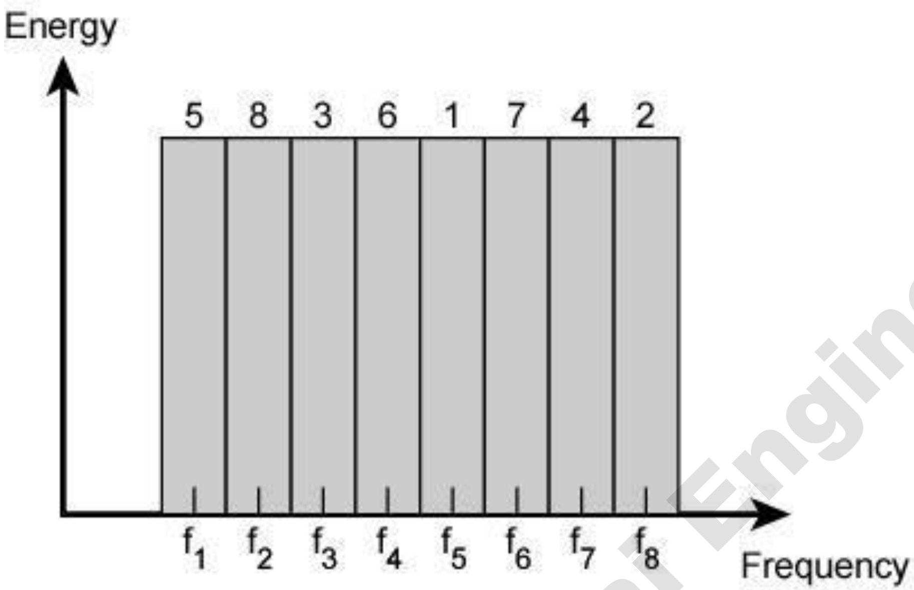
Frequency Hopping Spread Spectrum (FHSS)

- Signal broadcast over seemingly random series of frequencies
- Receiver hops between frequencies in sync with transmitter
- Eavesdroppers hear unintelligible blips
- Jamming on one frequency affects only a few bits

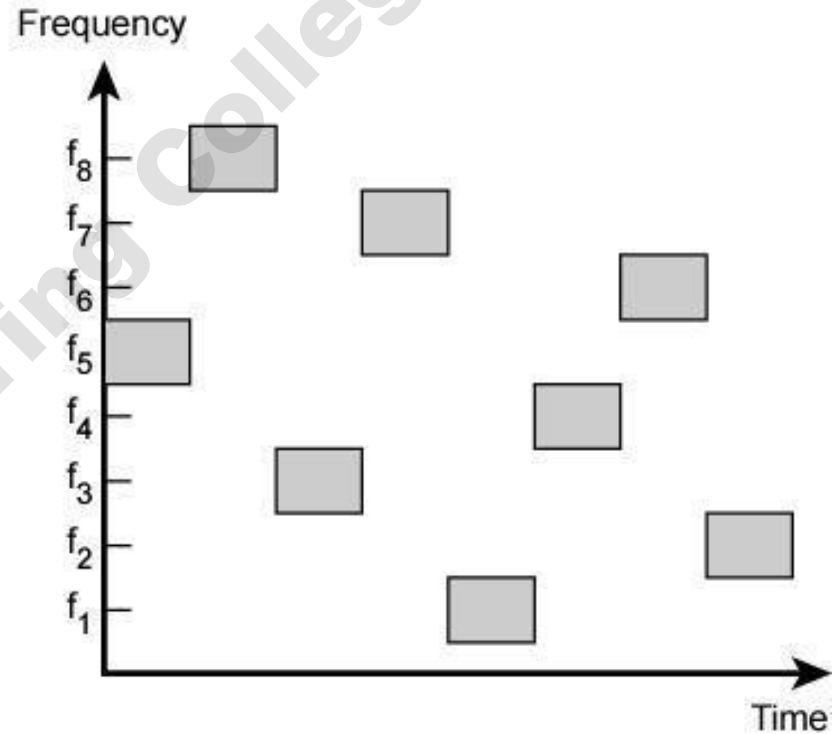
Basic Operation

- Typically 2^k carriers frequencies forming 2^k channels
- Channel spacing corresponds with bandwidth of input
- Each channel used for fixed interval
  - – 300 ms in IEEE 802.11
  - – Some number of bits transmitted using some encoding scheme
    - May be fractions of bit (see later)
  - – Sequence dictated by spreading code

Frequency Hopping Example

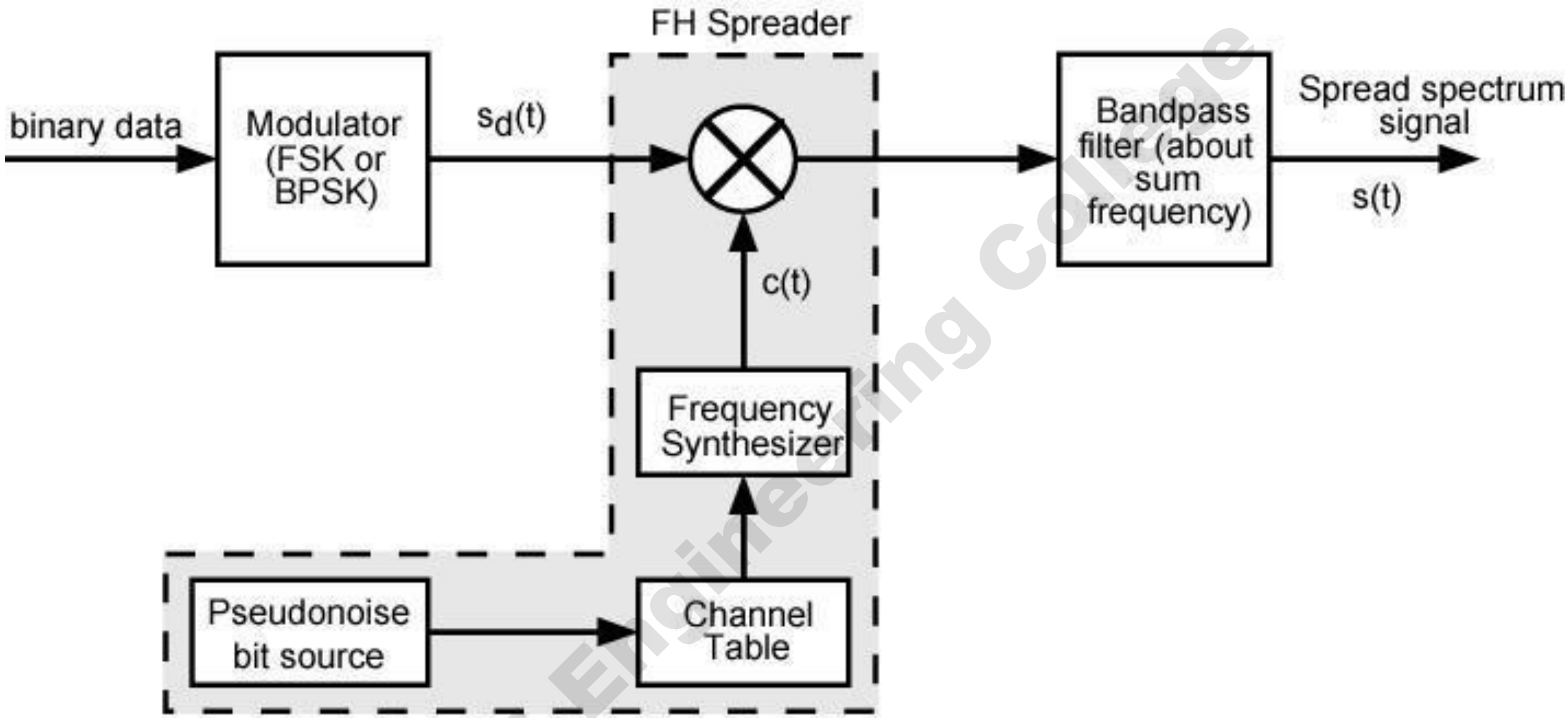


(a) Channel assignment

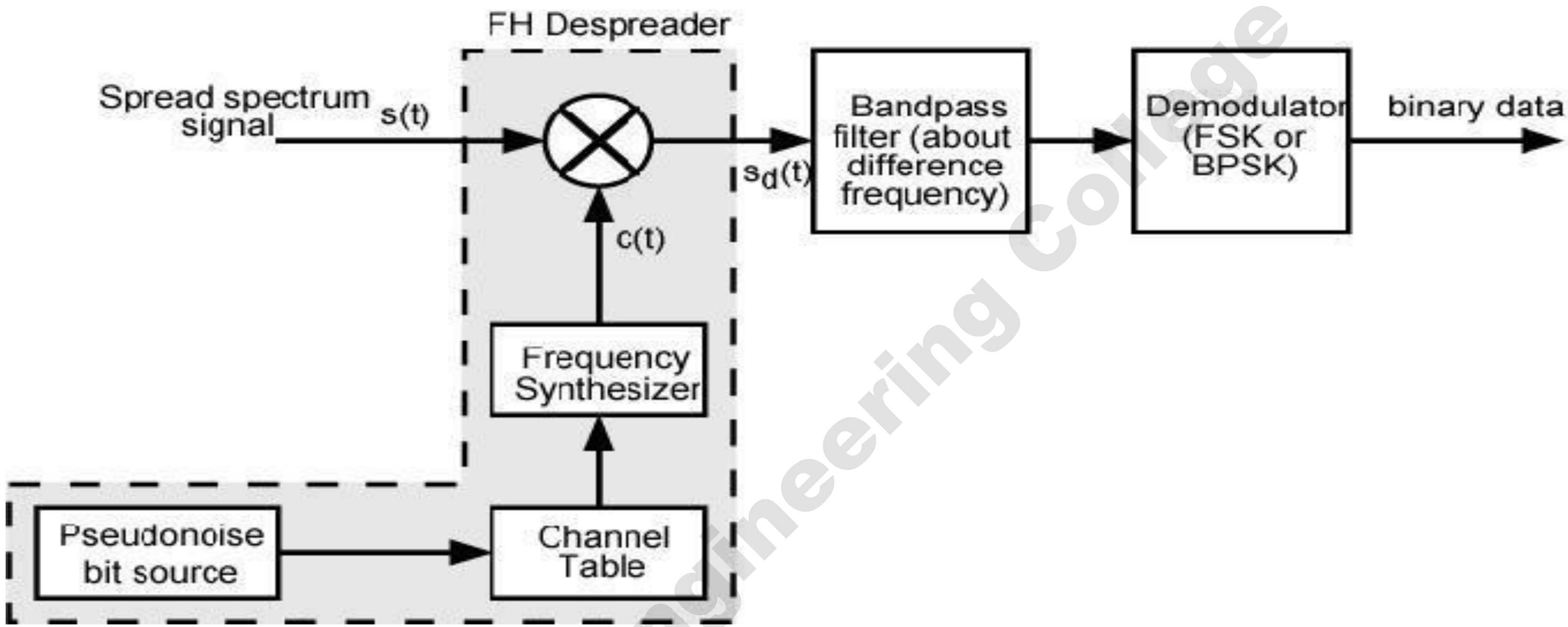


(b) Channel use

Frequency Hopping Spread Spectrum System (Transmitter)



Frequency Hopping Spread Spectrum System



Slow and Fast FHSS

- Frequency shifted every T_c seconds
- Duration of signal element is T_s seconds
- Slow FHSS has T_c □ T_s
- Fast FHSS has $T_c < T_s$
- Generally fast FHSS gives improved performance in noise (or jamming)

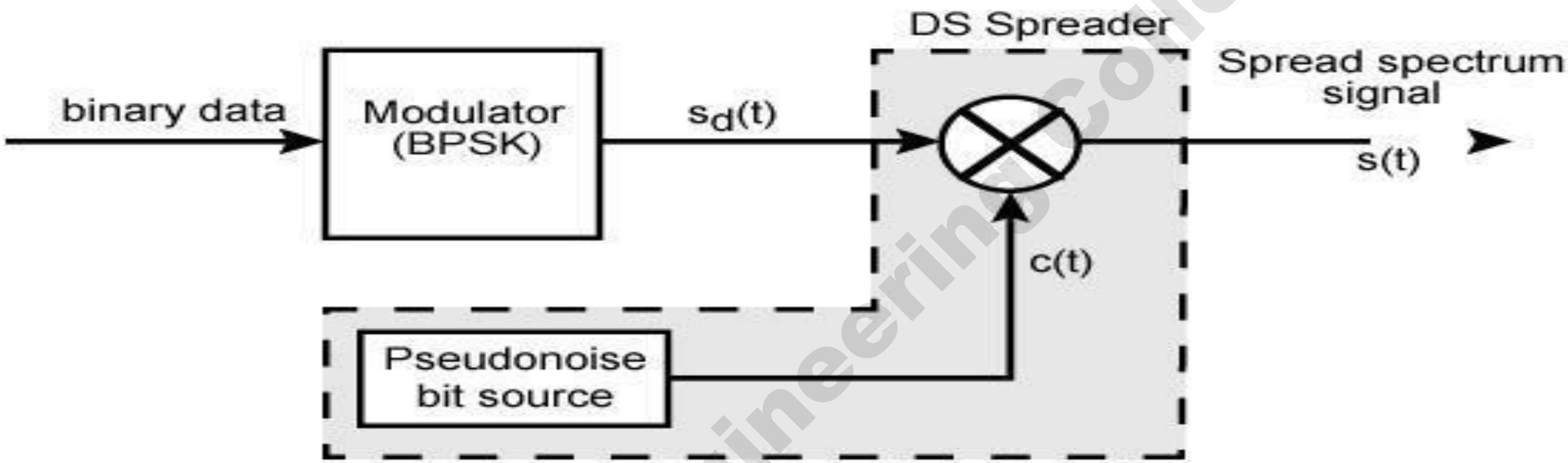
Direct Sequence Spread Spectrum (DSSS)

- Each bit represented by multiple bits using spreading code
- Spreading code spreads signal across wider frequency band
  - In proportion to number of bits used
  - 10 bit spreading code spreads signal across 10 times bandwidth of 1 bit code
- One method:
  - Combine input with spreading code using XOR
  - Input bit 1 inverts spreading code bit
  - Input zero bit doesn't alter spreading code bit

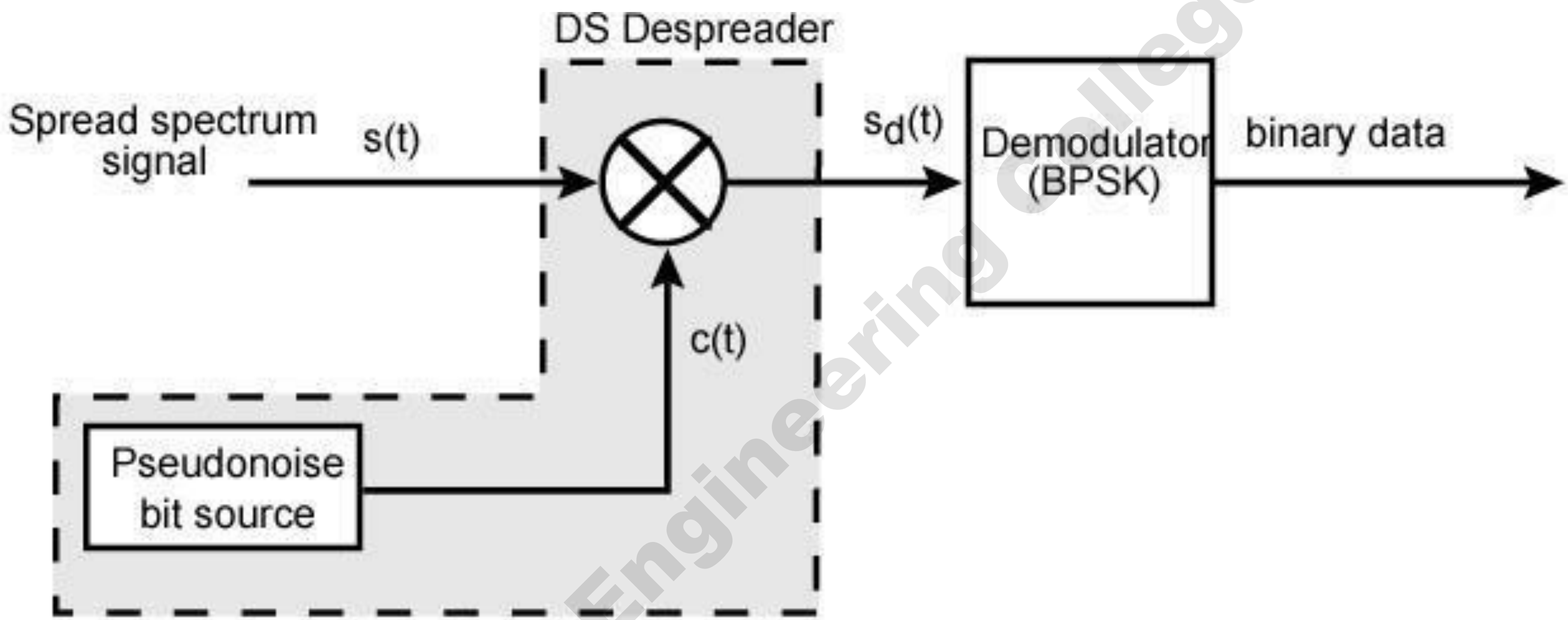
rate equal to original spreading code

- Performance similar to FHSS

Direct Sequence Spread Spectrum Transmitter



Direct Sequence Spread Spectrum Transmitter



Multiple Access Techniques

- The transmission from the BS in the downlink can be heard by each and every mobile user in the cell, and is referred as *broadcasting*. Transmission from the mobile users in the uplink to the BS is many-to-one, and is referred to as multiple access.
- Multiple access schemes to allow many users to share simultaneously a finite amount of radio spectrum resources.
  - Should not result in severe degradation in the performance of the system as compared to a single user scenario.
  - Approaches can be broadly grouped into two categories: narrowband and wideband.

Multiple Access Techniques

- Multiple Accessing Techniques : with possible conflict and conflict- free
  - Random access
  - Frequency division multiple access (FDMA)
  - Time division multiple access (TDMA)
  - Spread spectrum multiple access (SSMA) : an example is Code division multiple access (CDMA)
  - Space division multiple access (SDMA)

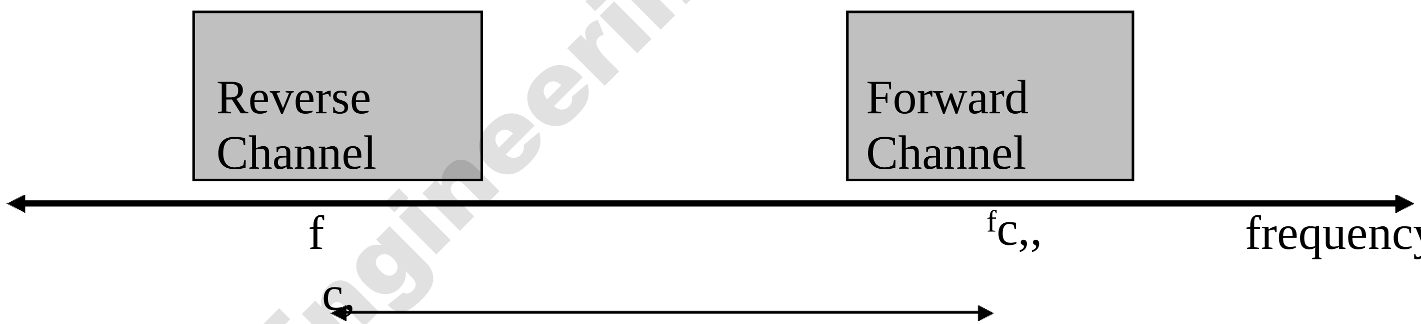
Duplexing

- For voice or data communications, must assure two way communication (duplexing, it is possible to talk and listen simultaneously). Duplexing may be done using frequency or time domain techniques.
  - Forward (downlink) band provides traffic from the BS to the mobile
  - Reverse (uplink) band provides traffic from the mobile to the BS.

frequency division duplexing (FDD)

- Provides two distinct bands of frequencies for every user, one for downlink and one for uplink.
- A large interval between these frequency bands must be allowed so that interference is

minimized.



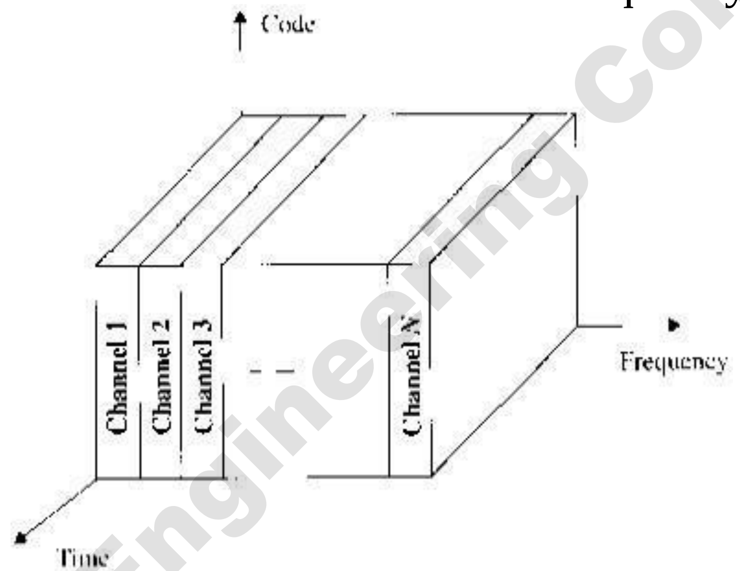
Frequency separation

Frequency separation should be carefully decided Frequency separation is constant

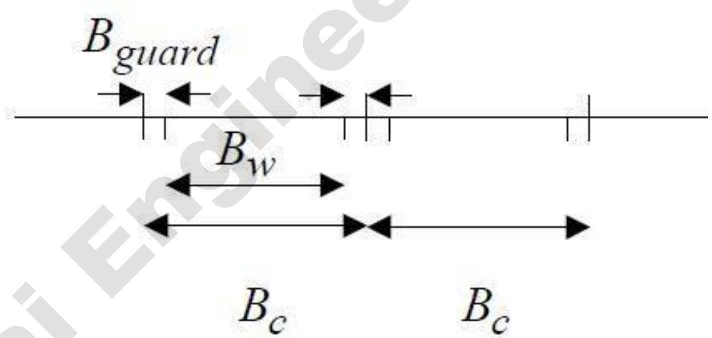
TDD

- In TDD communications, both directions of transmission use one contiguous frequency allocation, but two separate time slots to provide both a forward and reverse link.
- Because transmission from mobile to BS and from BS to mobile alternates in time, this scheme is also known as “ping pong”.
- As a consequence of the use of the same frequency band, the communication quality in both directions is the same. This is different from FDD.

- In FDMA, each user is allocated a unique frequency band or channel. During the period of the call, no other user can share the same frequency band.

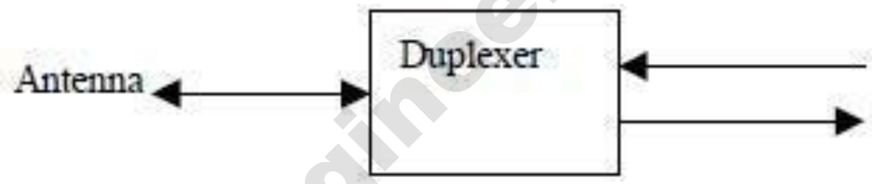


- All channels in a cell are available to all the mobiles. Channel assignment is carried out on a first-come first- served basis.
- The number of channels, given a frequency spectrum *BT* , depends on the modulation technique (hence *Bw* or *Bc*) and the guard bands between the channels 2*Bguard* . These guard bands allow for imperfect filters and oscillators and can be used to minimize adjacent channel interference.
- FDMA is usually implemented in narrowband systems.



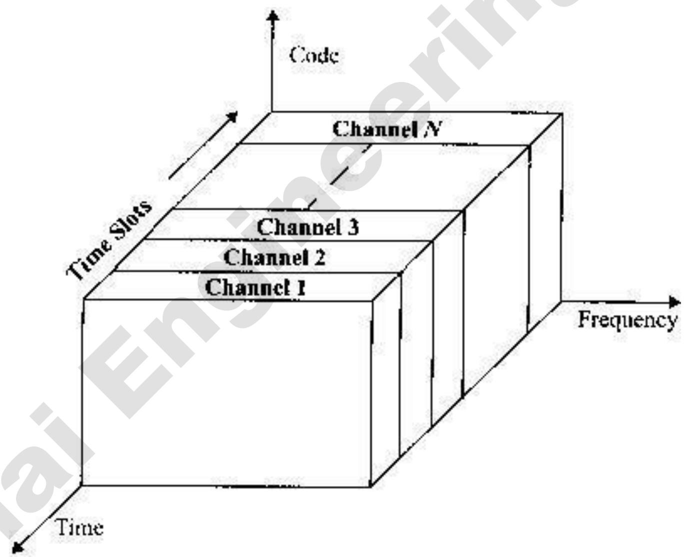
- Continuous transmission : the channels, once assigned, are used on a non-time-sharing basis. This means that both subscriber and BS can use their corresponding allotted channels continuously and simultaneously.
- Narrow bandwidth : Analog cellular systems use 25-30 kHz. Digital FDMA systems can make use of low bit rate speech coding techniques to reduce the channel band even more.
- If FDMA channels are not in use, then they sit idle and cannot be used by other users to increase capacity.
- Low ISI : Symbol time is large compared to delay spread. No equalizer is required (Delay spread is generally less than a few μs – flat fading).

- Low overhead : Carry overhead messages for control, synchronization purposes. As the allotted channels can be used continuously, fewer bits need to be dedicated compared to TDMA channels.
- Simple hardware at mobile unit and BS : (1) no digital processing needed to combat ISI (2) ease of framing and synchronization.
- Use of duplexer since both the transmitter and receiver operate at the same time. This results in an increase in the cost of mobile and BSs.



- FDMA required tight RF filtering to minimize adjacent channel interference.

- TDMA systems divide the channel time into frames. Each frame is further partitioned into time slots. In each slot only one user is allowed to either transmit or receive.
- Unlike FDMA, only digital data and digital modulation must be used.
- Each user occupies a cyclically repeating time slot, so a channel may be thought of as a particular time slot of every frame, where *N* time slots comprise a frame.

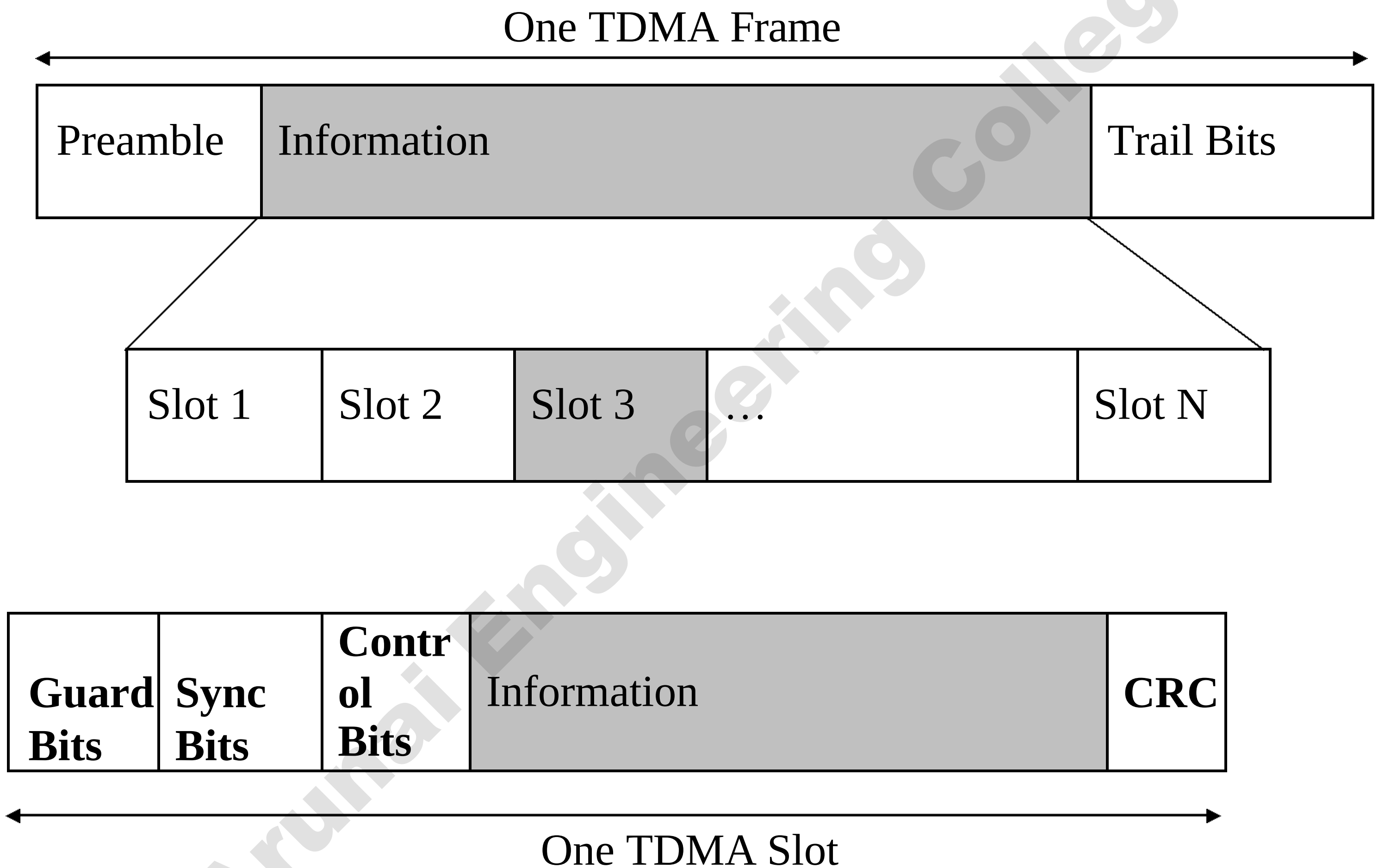


- Multiple channels per carrier or RF channels.
- Burst transmission since channels are used on a timesharing basis. Transmitter can be turned off during idle periods.
- Narrow or wide bandwidth – depends on factors such as modulation scheme, number of voice channels per carrier channel.
- High ISI – Higher transmission symbol rate, hence resulting in high ISI. Adaptive equalizer required.

Features

- High framing overhead – A reasonable amount of the total transmitted bits must be dedicated to synchronization purposes, channel identification. Also guard slots are necessary to separate users.

TDMA Frame



A Frame repeats in time

Code Division Multiple Access (CDMA)

- In CDMA, the narrowband message signal is *multiplied* by a very large bandwidth signal called spreading signal (code) before modulation and transmission over the air. This is called spreading.

- CDMA is also called DSSS (Direct Sequence Spread Spectrum). DSSS is a more general term.

- Message consists of symbols
  – Has symbol period and hence, symbol rate

Code Division Multiple Access (CDMA)

- Spreading signal (code) consists of chips
  - Has Chip period and and hence, chip rate
  - Spreading signal use a pseudo-noise (PN) sequence (a pseudo-random sequence)
  - PN sequence is called a codeword
  - Each user has its own cordword
  - Codewords are orthogonal. (low autocorrelation)
  - Chip rate is oder of magnitude larger than the symbol rate.
- The receiver correlator distinguishes the senders signal by examining the wideband signal with the same time-synchronized spreading code
- The sent signal is recovered by despreading process at the receiver.

CDMA Advantages

- Low power spectral density.
  - – Signal is spread over a larger frequency band
  - – Other systems suffer less from the transmitter
- Interference limited operation
  - – All frequency spectrum is used
- Privacy
  - – The codeword is known only between the sender and receiver. Hence other users can not decode the messages that are in transit
- Reduction of multipath affects by using a larger spectrum

CDMA Advantages

- Random access possible
  - Users can start their transmission at any time
- Cell capacity is not concerete fixed like in TDMA or FDMA systems. Has soft capacity
- Higher capacity than TDMA and FDMA
- No frequency management
- No equalizers needed
- No guard time needed
- Enables soft handoff